Magnetic confinement of high temperature plasma at the GOLEM tokamak

Abstract: This assignment introduces students to the physics, technology, diagnostics and operation of the GOLEM tokamak. It comprises performing a tokamak experiment and exploring the basic scaling properties of magnetic plasma confinement.

1 Tools

The GOLEM tokamak, basic GOLEM diagnostics (U_l wire, B_t coil, Rogowski coil, photodiode with H_α filter), RIGOL oscilloscope, device for remote tokamak operation (e.g. laptop).

2 Tasks

1. Homework before the measurements: Learn how to access and manipulate the remote data files from GOLEM measurements according to [1]. Then use the link at [1] to access the web-based virtual control room and get familiar with the web interface. At least one group member must have the remote file access already set up and tested when the experiment begins.
2. Connect to the web control interface of your group's oscilloscope (the link will be provided to you on the spot) and set up your oscilloscope for measurement. Use [2] as a reference.
3. Using the remote control room, execute a test tokamak discharge with arbitrary values of U_B, U_{CD} and p_0, pre-ionisation on. Plot the time traces of the individual diagnostic signals:
    - loop voltage U_l (channel 1)
    - voltage induced on the small B_t measuring coil (channel 2)
    - voltage induced on the Rogowski coil (channel 3)
    - voltage of the photodiode with an H_α filter measuring the plasma radiation intensity (channel 4)
4. Using the remote control room, execute 10 discharges with 5 different values of U_B, 2 different values of U_{CD}, arbitrary but constant p_0 and T_{CD} and pre-ionisation on. This discharge series may be shared between the present groups so that the laboratory as a whole takes less time.
5. Process the oscilloscope data as described in sections 5 and 6. Compare standard GOLEM diagnostics output with processed oscilloscope data for one discharge.
6. For each of the discharges, calculate the energy confinement time τ_E and the toroidal magnetic field B_t during the quasi-stationary discharge phase. Plot a (B_t, τ_E) scatterplot with the errorbars representing the standard deviation (see section 6.2). Calculate the mean confinement time τ_E and compare it to the Neo-Alcator scaling law [3, page 1131], which relates the ratio of the confinement time τ_E and the electron density n_e to the tokamak major radius R and minor radius a.

3 Theoretical introduction

3.1 Tokamak purpose and principle

Tokamaks are machines with a strong magnetic field whose mission is to, one day, become fusion reactors fuelling clean and safe power plants. The basic task of a tokamak reactor is to heat and confine its fuel, a 50:50 mixture of deuterium and tritium, allowing thermonuclear fusion reactions to take place. These reactions generate heat (14.1 MeV per reaction) which is subsequently converted into electricity via the standard steam-turbine cycle.

One of the main challenges in current tokamak research is to confine the burning fuel. Although the fuel is very thin (its density is 5-8 orders of magnitude lower than the density of air), its temperature is extremely high, up to $\sim$100 million K. This is to ensure that when deuterium and tritium nuclei collide, they have sufficient energy to overcome the repulsive electrostatic barrier and fuse, hence *thermonuclear* fusion. Such high temperatures mean that the fuel is in the state of plasma, a collection of ionized nuclei and free electrons, and also that it must never directly touch the reactor walls. (For one, the plasma would cool down and cease to exist; for two, the reactor walls might melt.) Tokamaks confine the fuel using the Lorentz force $q\mathbf{v} \times \mathbf{B}$, which forces charged particles to rotate around magnetic field lines rather than travel across them freely. Thus the strong magnetic field confines the plasma in the centre of the tokamak chamber.

The basic structure of a tokamak is shown in figure 1 (more information can be found e.g. in [4]). Tokamaks comprise three essential parts: a vacuum chamber, toroidal field coils, and a transformer. The *vacuum chamber* (or vacuum vessel) has the shape of a torus of the size approximately 1-20 m across; its purpose is to contain the plasma while allowing limited access through diagnostic ports. Around the vacuum chamber are wrapped dozens of *toroidal magnetic field coils*, which generate the confining toroidal magnetic field B_t (0.5-5 T). Finally, the *transformer* creates

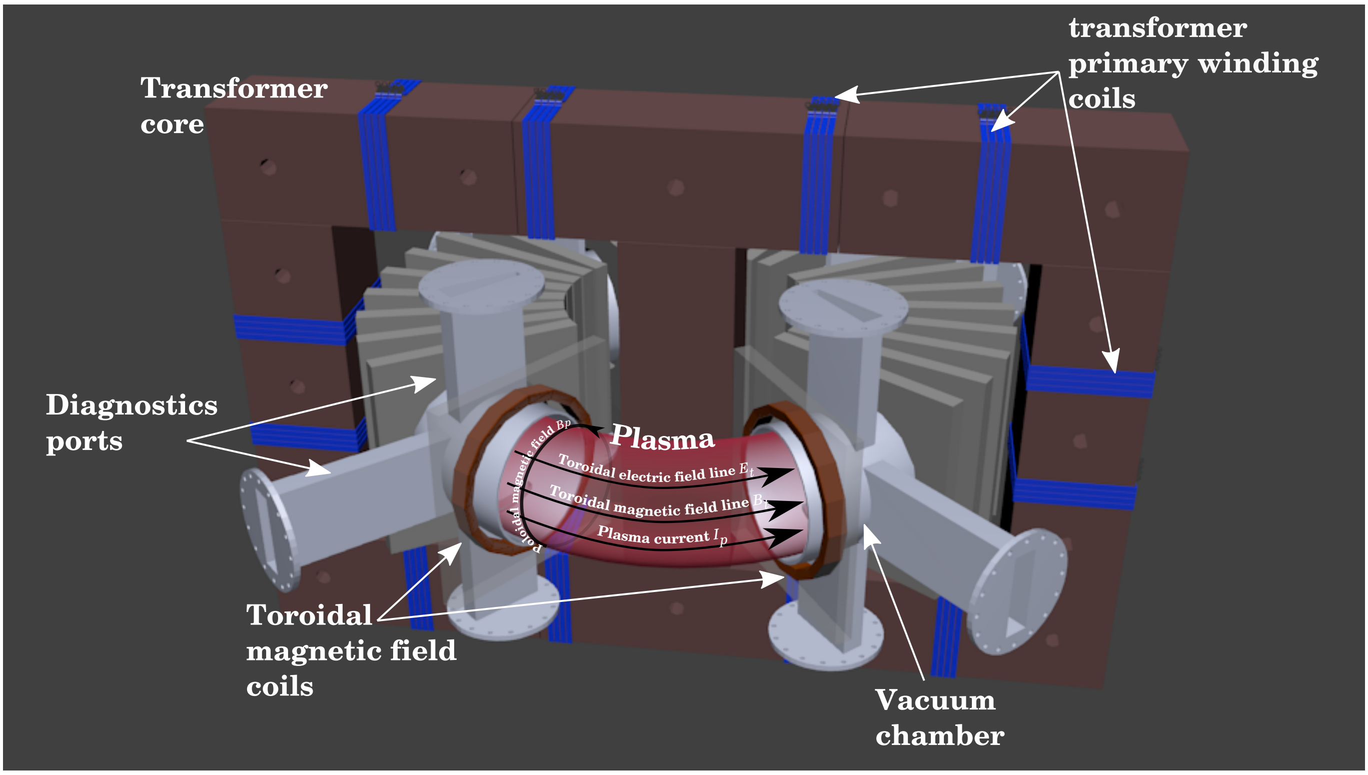

Figure 1: Basic components of the GOLEM tokamak.

and heats the plasma by inducing a loop voltage U_l (several V) inside the vacuum chamber and then driving a plasma current I_p (kA to MA).

Because of this structure, the duration of tokamak plasma existence is intrinsically limited. The plasma can only exist so long as the plasma current I_p is driven, because the ohmic heating $P_{OH} = U_l.I_p$ sustains its high temperature in spite of continuous heat losses. (It also ensures plasma stability, but that is outside the scope of this manual.) And since driving a current in the secondary coil (plasma) requires a monotonically changing current in the primary coil (shown in figure 1), which cannot be done forever, at some point the primary coil current reaches a maximum and the transformer stops transforming. Presently the plasma current dies out, the plasma cools down, electrons and ions recombine into a neutral gas and the plasma ceases to exist. Therefore, tokamak plasmas are created in so called *discharges*, or *shots* for short. Discharge duration strongly depends on the machine — on GOLEM it is < 20 ms, on the largest machines it is > 1 s.

3.2 Parameters of the GOLEM tokamak

The GOLEM tokamak is a rather small machine with a low magnetic field and a (relatively to other tokamaks) low plasma temperature. Its major radius (distance from the machine centre to the vessel centre) is $R = 40\,\mathrm{cm}$ and its minor radius (distance from the plasma vessel centre to the limiter, and therefore the maximum plasma column radius) is $a = 8.5\,\mathrm{cm}$. The resulting plasma volume is approximately $V_p = 80$ l. Its toroidal magnetic field B_t can rise up to 0.5 T and its plasma current I_p can reach 8 kA. The resulting electron density n_e is of the order of $10^{18}\,\mathrm{m}^{-3}$,[1] while the electron temperature T_e can reach several tens of electronvolts[2].

3.3 Theory of plasma energy confinement

Confining the energy stored in the plasma is a tokamak's prime duty. The fewer losses are allowed (via radiation, plasma particles escaping the confinement etc.), the better the tokamak. The energy stored in the plasma may be approximated as

$$W_p = \frac{1}{3} e n_e T_e V_p \tag{1}$$

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[1] In the context of tokamaks, density is given in the number of particles per cubic metre (m^{-3}).

[2] In the context of tokamaks, plasma temperature is typically given in electronvolts (eV), which is a unit of energy, not temperature. 1 eV is the energy gained by an electron by traversing a potential fall of 1 V. That may not seem like much, but when the typical kinetic energy of gas particles is 1 eV, the gas temperature is 11 600 K. To make matters more confusing, plasma temperature is sometimes given in eV, sometimes in J, and sometimes in K. The conversion is T [J] $= k_B T$ [K] $= k_B T/e$ [eV], where k_B is the Boltzmann constant and e is the elementary charge. Within this manual, plasma temperature is consistently given in eV.

where e is the elementary charge [C], n_e is the electron density [m^{-3}], T_e is the central electron temperature [eV], V_p is the plasma volume [m^3] and the resulting energy W_p is in joules. The equation is derived assuming equal shares of energy between ions and electrons, a parabolic profile in electron temperature and only 2 degrees of freedom (a consequence of the magnetic geometry). During a tokamak discharge, this energy is continually lost and the losses must be fully replenished by heating the plasma, $P_{heating} \approx P_{loss}$. If the heating is suddenly turned off, the plasma will start losing energy exponentially, $W_p = W_{p0}.\mathrm{e}^{-t/\tau_E}$, as illustrated by figure 2. The slower the energy decay is, the better the plasma energy is confined. The confinement quality is therefore characterised by the quantity τ_E, which is called the *energy confinement time*, or just the confinement time.[3] The higher the confinement time is, the better the tokamak.

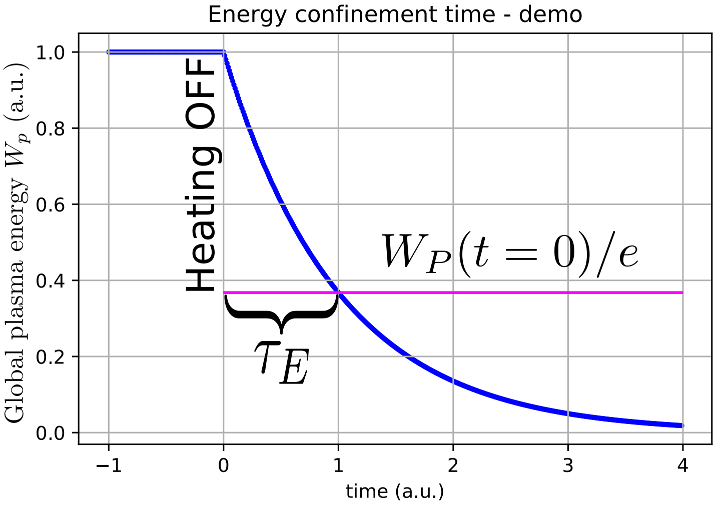

Figure 2: Time evolution of total plasma energy W_p after heating is turned off, showing the meaning of the energy confinement time τ_E.

3.4 Energy confinement time measurement

Your task at GOLEM is to measure its confinement time τ_E and to see whether it depends on the toroidal field magnitude B_t. (Hint: it should.) The trick to the calculation of τ_E is that energy is lost from the plasma regardless of whether it is being heated or not. The loss power is always $P_{loss} = W_p/\tau_E$. Therefore, in the quasi-stationary phase of the GOLEM discharge

$$P_{heating} = P_{loss} = W_p/\tau_E. \tag{2}$$

There is only one source of heating at GOLEM, and that is the ohmic heating $P_{heating} = U_l I_p$. Thus, the energy confinement time τ_E may be calculated by combining equations (2) and (1) as

$$\tau_E = \frac{e n_e T_e V_p}{3 U_l I_p}. \tag{3}$$

You will measure U_l, I_p and B_t using your own set of diagnostics (a loop voltage coil, a Rogowski coil and a B_t coil) plugged into an oscilloscope. The electron density n_e and the central electron temperature T_e can be estimated in the following fashion.

3.5 Electron density measurement

To estimate the mean electron density n_e inside the tokamak, suppose that the gas inside the tokamak chamber is pure hydrogen H_2 and that in plasma state it is completely ionised. Consequently, the number of electrons in the plasma N_e is the same as the number of neutral hydrogen atoms in the gas N_0, which is double the amount of neutral hydrogen molecules.

$$N_e = N_0 = 2N_{H_2}$$

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[3] Note that the confinement time tells you nothing about discharge duration! Of course, machines with better plasma confinement are generally able to perform longer discharges, but the actual number of seconds can be very unrelated. Typically τ_E is orders of magnitude shorter than the discharge duration.

Assuming that all the electrons are trapped inside the plasma volume V_p, the average electron density is $n_e = \frac{N_e}{V_p}$. The number of hydrogen molecules can be calculated from the neutral gas pressure p_0 and temperature T_0 via the ideal gas state equation:

$$p_0 V_{ch} = N_{H_2} k_B T_0$$

where $V_{ch} = 150$ l is the entire chamber volume (not just the plasma volume since the neutral gas fills the whole chamber), $k_B = 1.38 \times 10^{-23}$ JK^{-1} is the Boltzmann constant and $T_0 = 300$ K is the room temperature. Putting all of these formulas together, one has a rough estimate of the electron density:

$$n_e = \frac{2 p_0 V_{ch}}{k_B T_0 V_p} \tag{4}$$

3.6 Central electron temperature measurement

The central electron temperature of a pure hydrogen plasma is given by Spitzer's formula [5]:

$$T_e = 0.9 \cdot R_p^{-\frac{2}{3}} \tag{5}$$

where R_p [Ω] is the plasma resistivity. Reverting the equation, one finds that the plasma resistivity falls as T_e increases; in contrast, metal resistivity increases with temperature. This is because in metals, increased temperature means stronger vibration of the atomic lattice, which hinders the conducting electrons, while in plasmas increased electron velocity lowers the Coulomb interaction cross-section, decreasing the friction. To calculate the plasma resistivity, one simply uses Ohm's law for the plasma circuit: $U_l = R_p I_p$. (This applies only in the stationary phase of the discharge, where inductivity effects can be neglected.)

4 Experimental setup

4.1 Technological details of the GOLEM tokamak

The basic tokamak features described in section 3.1 can be implemented in a number of different ways, depending on the machine size and budget. The toroidal field coils, for example, can be superconducting (cooled by liquid helium), cryogenic (cooled by liquid nitrogen to reduce the resistance and losses via ohmic heating), water-cooled, or air-cooled (meaning not actively cooled). In this section we shall describe two particular properties of tokamak GOLEM: its capacitor power supply and its vacuum system.

The GOLEM discharge power is supplied by two *capacitors banks* (one for the toroidal magnetic field circuit and one for the primary transformer winding circuit), which are stored in a separate room below the tokamak. During a discharge, significant currents must be driven through the 28 toroidal field coils and the transformer primary circuit in order to generate sufficient toroidal magnetic fields and plasma current. The necessary power is too large to be drawn directly from the electric network, and so GOLEM employs capacitors. Prior to the discharge, the capacitors are charged to the requested voltages U_B and U_{CD}[4], respectively. At the discharge beginning, computer-controlled thyristors connect them to their two respective circuits (see figure 3); either simultaneously, or with a variable time delay T_{CD}. This results in the typical sine-like time evolution of the toroidal field B_t and the plasma current I_p (see figure 5), as the capacitors discharge freely into the two respective circuits.

Tokamak operation always requires a reliable vacuum system. Fusion reactors will use a deuterium-tritium mixture, but most current tokamaks use the cheaper hydrogen, deuterium or helium as a similar-acting substitute. Plasma properties significantly depend on the plasma composition; even a small percentage of impurities (such as carbon, nitrogen, oxygen or water) can drastically change the tokamak performance. For instance, a pure hydrogen plasma glows pink, but GOLEM "hydrogen" plasma glows blue. The basic way to control the impurity content is to continuously evacuate the vacuum chamber to ≈ 0.2 mPa and only fill it up with working gas shortly before the discharge. The GOLEM tokamak has two working gases: hydrogen and helium (deuterium is more expensive and we don't really need it). In this laboratory assignment, only hydrogen is used. Its initial pressure p_0 is one of the discharge parameters which must be set for every discharge.

4.2 GOLEM discharge setup

To set up a GOLEM discharge in the remote control room [6], the user must specify the following parameters (in this order):

- **Initial neutral gas pressure** p_0**.** The recommended values are 15-40 mPa. The influence of initial gas pressure on plasma quality is complicated. Too low a pressure will result in a vacuum discharge (there is too little gas to make plasma from). Too high a pressure will also result in a vacuum discharge (the mean free path is too short

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[4] CD = Current Drive.

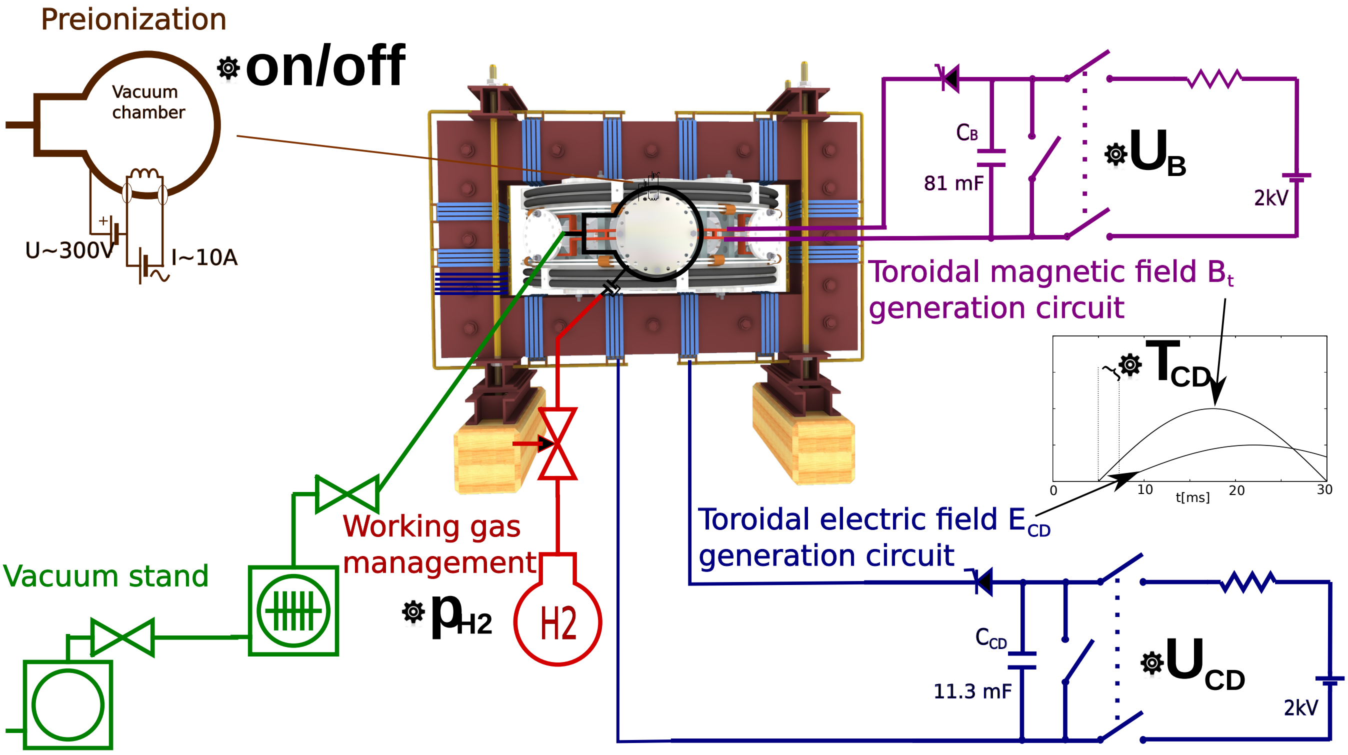

Figure 3: Experimental set-up: schematic of the GOLEM tokamak with its various sub-systems.

for charged particles to gain sufficient energy in the accelerating loop voltage to ionise another particle, so the avalanche breakdown into a plasma fails; see Paschen's law). You can experiment with it a little.

- **Working gas type.** Use hydrogen.
- **Use/not use pre-ionisation.** Plasma is created in the vacuum chamber thanks to the loop voltage induced by the transformer. This voltage accelerates the existing charged particles in the chamber until they gain sufficient energy to ionise a neutral gas particle. The resulting ion-electron pair are also accelerated by the loop voltage, causing an avalanche ionisation which eventually turns the neutral gas into a plasma. Turning pre-ionisation on helps this process by providing a larger number of initial charged particles, namely electrons which are emitted from a heated tungsten filament inside the chamber. In effect, if you want a "vacuum discharge" (a discharge where plasma was not created despite the existing loop voltage and magnetic field), turn pre-ionisation off. Otherwise leave it on.
- **Voltage U_B on the capacitor bank powering the toroidal field coils.** The bank capacity is $C_B = 81$ mF. The recommended values of U_B are 800-1000 V. The higher the voltage, the higher the toroidal magnetic field.
- **Voltage U_{CD} on the capacitor bank powering the transformer primary coils.** The bank capacity is $C_{CD} = 11.3$ mF. The recommended values of U_{CD} are 400-600 V. Typically the higher the voltage, the higher the plasma current. However, plasma current depends strongly on other factors (plasma purity, current toroidal magnetic field, initial pressure etc.), so this rule isn't very reliable.
- **Time delay T_{CD} between discharging the two capacitor banks.** Typically B_t takes a longer time to reach its maximum value than I_p. Therefore, it may be desirable to switch the transformer circuit on *after* the toroidal field coil circuit. T_{CD} gives the time delay in microseconds. The recommended value is 0-10000 μs.
- **Discharge comment.** An eloquent comment is important for data processing, as it allows quick retrospective identification of the discharges. Examples:

bad comment	good comment
`(empty)`	`PRA2 group6 test`
`test`	`PRA2 sk6 UB=800, UCD=1000, TCD=0, p=20`
`UB=800, UCD=1000, TCD=0, p=20`	`PRA2 group6 vacuum discharge`

Finally, note that we recommend some values because they maximise the chance of getting a nice plasma discharge, not because values outside these intervals aren't safe for the machine. The only downside of a failed discharge is that it takes 2-3 minutes to repeat it, which might be a lot during the laboratory.

5 GOLEM tokamak diagnostics

5.1 Diagnostics overview

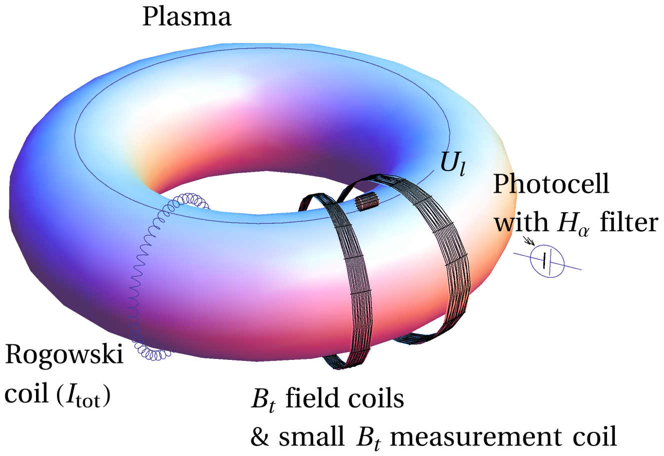

Figure 4: Tokamak diagnostics used in this assignment.

In tokamak research, the plasma is closely watched and controlled by a set of diagnostics. Figure 4 shows four basic tokamak diagnostics:

- A wire loop laid toroidally along the plasma ring: measures the **loop voltage** U_l.
- A small coil attached to the vessel, its axis in the toroidal direction: measures the time derivative of the **toroidal magnetic field** dB_t/dt.
- A Rogovski coil tied around the vessel: measures the time derivative of the total poloidal magnetic field dB_p/dt. The poloidal field consists of two contributions: (i) the field generated by the **plasma current** I_p, and (ii) the field generated by the current I_{ch} running through the tokamak chamber, induced by the loop voltage along with the plasma current.
- A photodiode with an H_α filter: measures the radiation intensity of the H_α spectral line (a dominant line in the hydrogen spectrum).

An example of the time evolution of these quantities is shown in figure 5, with the addition of the line-averaged electron density measured by an interferometer. The quantities marked in the bold face are the ones you will process for each discharge to calculate the plasma parameters. Each of the three diagnostics, the wire loop, the measuring coil, and the Rogowski coil, has its own particular signal processing. This is explained in the following sections.

5.2 Wire loop

The wire loop signal requires no post-processing (beside offset removal if needed; see section 6). The loop voltage U_l is the direct output of channel 1 measurement.

5.3 B_t measuring coil

According to Faraday's law of induction, if the magnetic flux passing through a conductive loop changes, a voltage U is induced on it. Assuming that the loop is small so the magnetic field inside it is uniform, the voltage magnitude is

$$U = NS\frac{dB_\perp}{dt} \tag{6}$$

where N is the number of the coil threads ($N = 1$ for a single loop), S is the loop area and $B_\perp$ is the magnetic field component perpendicular to the loop area.

Electromagnetic induction is also the principle of the B_t loop measurement: the coil is simply placed into the magnetic field, its axis pointing along the toroidal direction, and its signal (the voltage U_{B_t}) is integrated in time and calibrated. The calibration constant is theoretically equal to NS; however, in this assignment you will calibrate the signal by comparing the toroidal magnetic field B_t measured by the standard GOLEM diagnostic set to your own measurements of $\int_0^t U_{B_t}(\tau)d\tau$.

$$B_t(t) = C_{B_t}\int_0^t U_{B_t}(\tau)d\tau \tag{7}$$

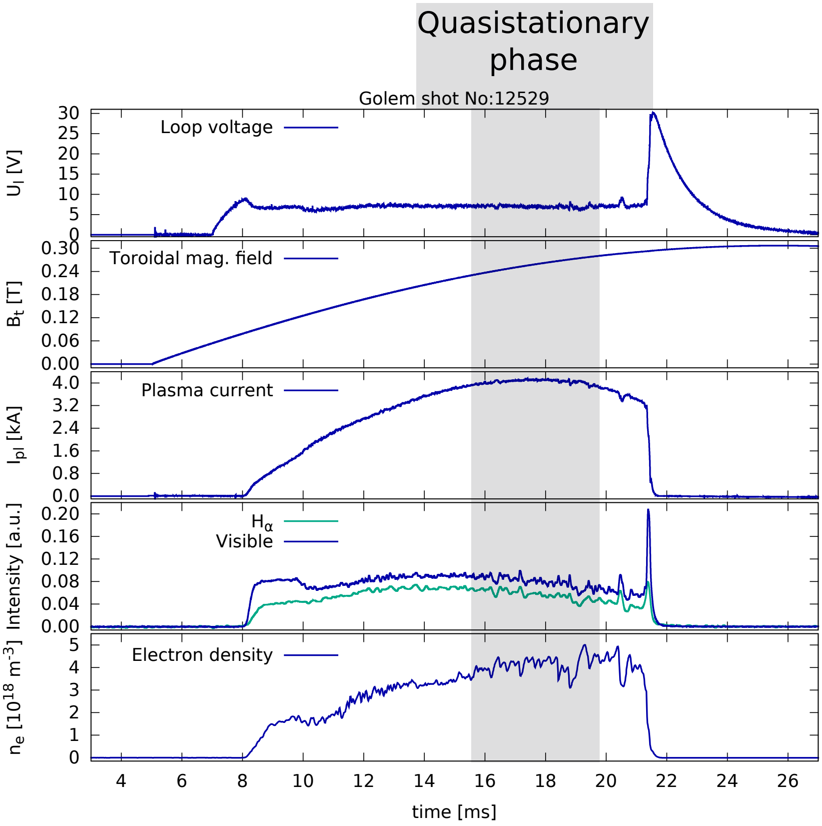

Figure 5: Time evolution of a well executed GOLEM discharge. From top to bottom - loop voltage U_l, toroidal magnetic field B_t, plasma current I_p, H_α spectral line intensity and line-averaged electron density n_e.

This calibration may be done individually for every individual discharge, but it is more convenient to calculate the calibration constant C_{B_t} once and then reuse it. (Note, however, that every B_t coil is placed differently and so calibration constants of distinct coils will be different.)

5.4 Rogowski coil

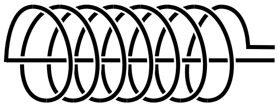

Figure 6: Rogowski coil scheme.

The Rogowski coil is the most complicated of the three self-implemented diagnostics whose signal you will post-process. It is a "coil loop" — a one-metre long thin coil which is wrapped around the tokamak chamber poloidally. As seen in figure 6, one of the Rogowski coil ends is directly accessible, while the other leads through the coil to negate the toroidal magnetic field contribution in its signal. As a result, the coil only picks up the poloidal magnetic field via electromagnetic induction,

$$U_{RC} \propto \frac{dB_p}{dt}.$$

The poloidal magnetic field has two components: the field $B_{p,p}$ generated by the plasma current I_p and the field $B_{p,ch}$ generated by the toroidal current I_{ch} induced by the loop voltage in the tokamak chamber. The respective currents are then proportional to their magnetic field according to the Biot-Savart law. The chamber current contribution is unwanted and has to be removed in order to find the plasma current I_p. Luckily, I_{ch} can be easily calculated using the loop voltage and the chamber resistivity, $I_{ch}(t) = U_l(t)/R_{ch}$ where $R_{ch} = 0.0097\,\Omega$. The calibration constant C_{RC} can then be defined with the relation

$$I_p(t) + \frac{U_l(t)}{R_{ch}} = C_{RC} \int_0^t U_{RC}(\tau)d\tau \tag{8}$$

and calculated from the standard diagnostic output on the left-hand side and the oscilloscope data on the right-hand side. With C_{RC} and R_{ch} known, the plasma current can finally be calculated from the oscilloscope data as

$$I_p(t) = C_{RC} \int_0^t U_{RC}(\tau) d\tau - \frac{U_l(t)}{R_{ch}}. \tag{9}$$

6 Technical details of signal processing

6.1 Offset removal

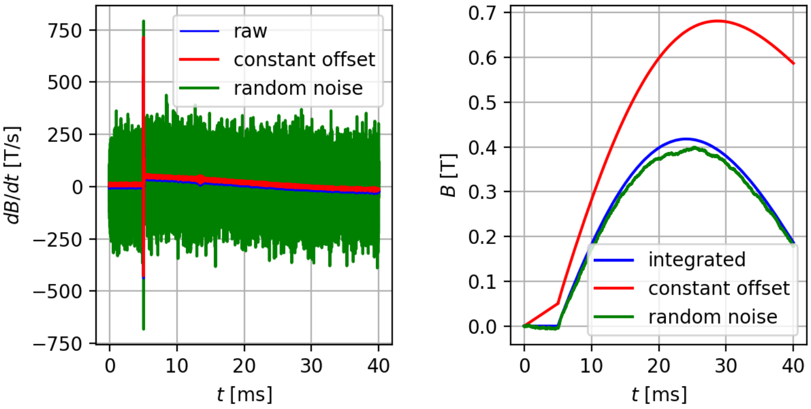

Figure 7: Time traces of dB/dt and integrated B in GOLEM discharge #31150, demonstrating the offset impact on the integrated data quality.

Magnetic measurements via electromagnetic induction are quite simple to set up, but their disadvantage lies in the need to integrate the signal. This involves many problems, the most basic of which is handling the *offset*. An offset is a non-physical addition to a signal, created by noise in the electronics, parasitic voltages, cross-talk between the diagnostics and many other influences. Figure 7 demonstrates the impact an offset can have on data integration. In blue, the real, physical dB/dt and B are plotted. In red, a small constant offset was added to the entire raw signal. In green, a normal-distributed noise with a zero mean was added to the raw signal. One observes the effect that this has on the integrated signal, in particular the *drift* in the red case.

In this assignment, offsets are more than likely to appear on the U_l, U_{B_t} and U_{RC} signals. The simplest method to remove them is to average the first few hundred/thousands of samples (may vary depending on oscilloscope trigger settings) and subtract this average from the signal prior to its integration. This method will fail if the offset is time-dependent, but in that case removing it is a whole new problem which cannot be addressed within the time frame of this assignment.

6.2 Signal processing, averaging and error propagation

The signal processing workflow is shown in figure 8. After the red-marked quantities are calculated, they are averaged over the quasi-stationary part of the discharge (see figure 5) and their error is taken as the standard deviation within the same time window. The electron temperature T_e and the energy confinement time τ_E are then calculated from these mean values and their uncertainties are estimated using the standard error propagation tools. This means that we neglect the errors contained in the calibration constants, the chamber current I_{ch} and the offset removal.

7 Acknowledgments and feedback

The invaluable effort of the following individuals in preparing this exercise is deeply appreciated (alphabetic order): Ondřej Ficker, Ondřej Grover, Remy Guirlet, Kateřina Jiráková, Jaroslav Krbec, Gergo Pokol, Jan Stöckel and Milos Vlainic.

If you, the reader, wish to add your name to this list, send us your feedback to `svoboda@fjfi.cvut.cz`. Especially useful and constructive feedback will be appropriately rewarded.

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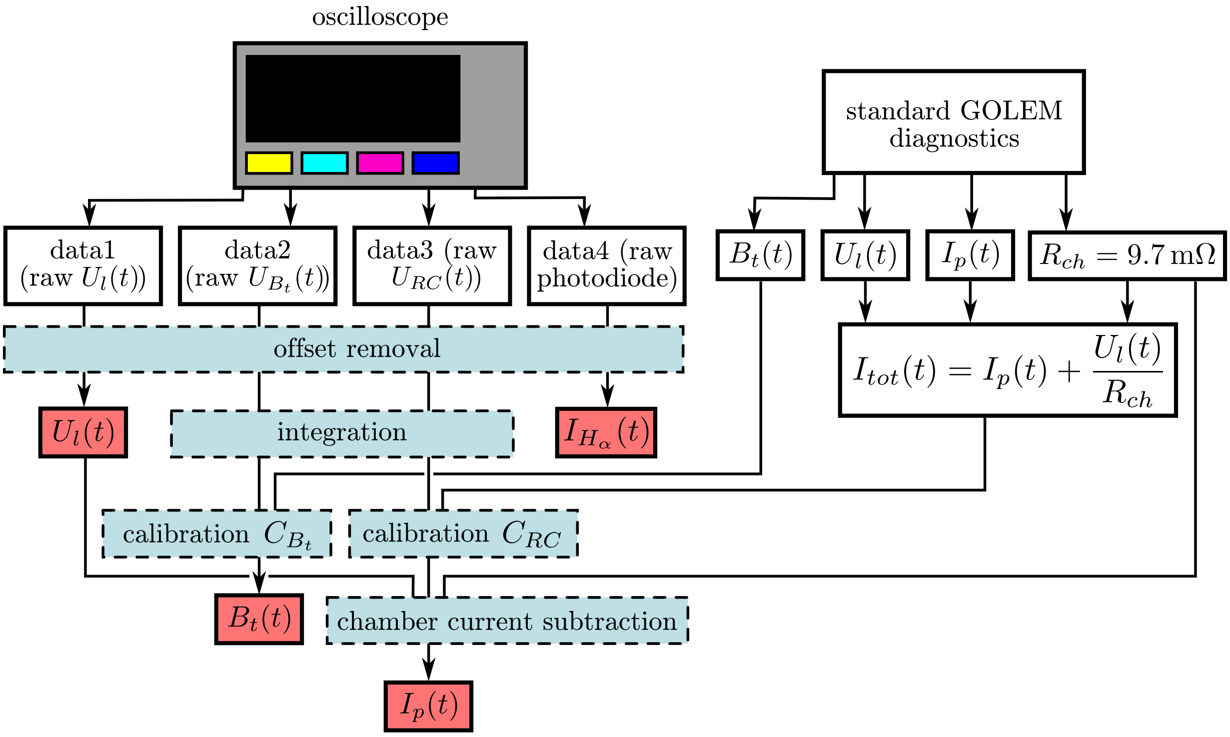

Figure 8: Flow chart of the signal processing procedure.

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