# PART 2: TASKS

## 1 Characterisation of Plasma with Microwaves with Ana Kostić: kostic.ana.ka@gmail.com

In order to assess the viability of a tokamak reactor, it is necessary to diagnose the plasma. A large group of diagnostics is based on the analysis of radiation emerging from the plasma. This may concern either radiation emitted by the plasma itself or it may be radiation originating from an external source, transmitted through the plasma and affected by it. Microwave diagnostics play an important role in this respect.

Measuring techniques based on microwaves have been playing an important role in fusion plasma experiments. The most useful microwave diagnostic techniques can be divided into a number of categories:

- detection of **electron cyclotron emission** or synchrotron radiation, to obtain information on the temperature of the electrons in the plasma,
- study of **collective scattering** of incident waves to follow density fluctuations in plasmas,
- **interferometry** to measure plasma densities,
- **reflectometry** to determine the location of plasmas with specific densities, often combined with the measurement of fluctuations,
- **polarimetry** to measure the distribution of the magnetic field in the plasma.

Our focus for this hands-on experiment will be on the interferometry and the density measurements.

### 1.1 Basic Principle of Interferometry

Though the complete picture of the interaction of electromagnetic waves with plasmas is quite complicated, the description can be simplified considerably for tokamak plasmas: in the frequency range used here the ion may be considered to be infinitely heavy. In tokamak plasmas, the electron temperature is usually $\leq 10\,\text{keV}$. So the average thermal velocity of the electrons is much smaller than the velocity of the waves which is close to the velocity of light; consequently, relativistic effects are in general negligible. The electron-ion collision frequency is much lower than the frequency of the wave and, therefore, collisional damping effects can also be neglected.

The phase velocity of an electromagnetic wave traveling through plasma depends of the density: the index of refraction is $N(n_e) = c/v$ where $c$ is the velocity of light and $v$ the phase velocity of the electromagnetic wave in a plasma with an electron density $n_e$ . At a so-called critical density, $n_c$ , the phase velocity becomes infinite, so electromagnetic waves of a chosen frequency cannot propagate any more through the plasma (a phenomenon dubbed the cut-off) but are reflected.

Waves, when transmitted through a plasma, can be used to measure the electron density with interferometers, provided that a proper choice of the applied frequency is made as long as $n_e$ remains smaller than $n_c$. Interferometry is basically the measurement of the phase change due to the presence of the plasma.

If the plasma density is non-homogeneous and has a parabolic spatial distribution a wave passing through the plasma undergoes varying refractive effects all along its path and may therefore not travel along a straight line. The resulting angle of refraction depends on the chosen frequency, on the values of $n_e/n_c$, and on the spatial gradient of $n_e$ . For this reason the wavelength of the probing beam should be chosen far away from cut-off, i.e. as short as possible, but still within the range where the phase changes, due to the presence of the plasma, are still detectable. However, smaller wavelengths require a higher mechanical stability of the instrument to avoid spurious phase-effects due to vibrations. Thus, the choice of the operating frequency always implies a compromise and it also depends on the availability of microwave oscillators.

Single-beam interferometers are generally used. In most cases the microwave beam, with a limited angular spread, is directed parallel to the density gradient through the centre of the plasma, thus the influence of refractive effects is relatively small. Interferometry can provide density profiles when applied along a number of (parallel) chords. Since line-integrated densities are measured, we would have to unfold the interferometric information (e.g. by Abel inversion) to get the density profile. Multi-channel interferometers for large MCF experiments actually become physically dominating obstacles in the crowded space around a reactor and may also occupy many observational ports on the machine.

## 1.2 Principle of Plasma Density Measurements

An interferometer is in fact used to measure the difference in path length between a reference beam, which is kept constant in optical length, and a beam transmitted through the plasma. The optical length through the plasma varies because of the change in refractive index, connected with the increase and the decrease of the electron density, during the experimental sequence (Fig. 1)

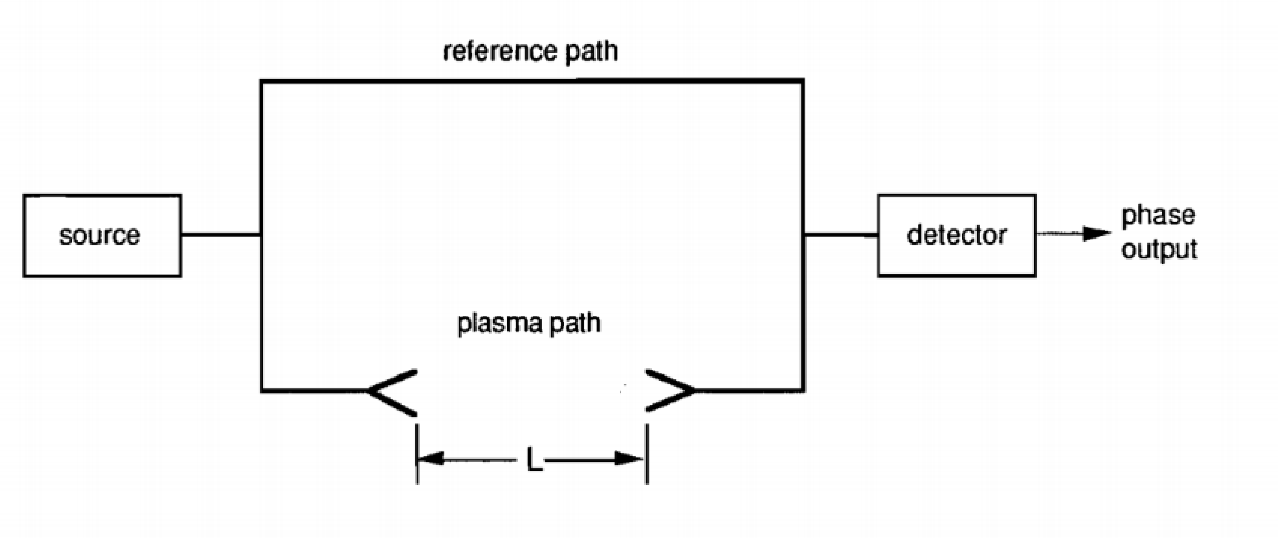


Figure 1: Mach-Zehnder interferometer.

The path length $L$ contains a certain number of vacuum wavelengths $\lambda_0$:

$$L = a\lambda_0, \text{ where } a = \frac{\varphi_0}{2\pi}. \quad (1)$$

But in the presence of plasma, the same path length will contain a certain number of plasma wavelengths $\lambda_p$:

$$L = b\,\lambda_p, \text{ where } b = \frac{\varphi_p}{2\pi}. \tag{2}$$

$\varphi_0$ and $\varphi_p$ are the phases of the wave after traveling through vacuum or through plasma, respectively. From the equations above, we can see that

$$\varphi_0 = \frac{L}{\lambda_0}2\pi \text{ and } \varphi_p = \frac{L}{\lambda_p}2\pi. \tag{3}$$

The refractive index is a dimensionless number that describes how electromagnetic waves propagates through a certain medium. Therefore it compares the speed of light in vacuum ($c$) with the phase velocity of the wave in that medium ($v$):

$$N = \frac{c}{v} = \frac{\lambda_0}{\lambda_p}. \tag{4}$$

When plasma is formed the detector (Fig. 1) will measure a phase change

$$\Delta\varphi = \varphi_0 - \varphi_p = 2\pi L\left(\frac{1}{\lambda_0} - \frac{1}{\lambda_p}\right) = \frac{2\pi L}{\lambda_0}(1-N). \tag{5}$$

Electromagnetic radiation in a magnetized plasma propagates in different modes, depending on direction of its wavevector $\vec{k}$, electric ($\vec{E}$) and magnetic field ($\vec{B}$) vectors with respect to the direction of an external magnetic field $\vec{B}_0$. To simplify this complicated picture, that would otherwise fill the entire book, let us take that $\vec{k}\cdot\vec{E} = 0$ and $\vec{E} \parallel \vec{B}_0$. In this case only a so-called O-wave (ordinary wave) propagates.

If the probing beam of the interferometer propagates through a magnetized cold plasma and we assume the orientation of the above vectors so that the O-wave propagates and, if we simplify the picture even more, the collisional effects can be assumed to be negligibly small, the refractive index can be simply formulated as:

$$N = \sqrt{1 - \frac{\omega_p^2}{\omega^2}}, \tag{6}$$

where $\omega$ is the frequency of the probing beam and $\omega_p$ is the electron plasma frequency

$$\omega_p = \sqrt{\frac{e^2 n_e(r)}{4\pi^2\varepsilon_0 m_e}} = 8.98\sqrt{n_e(r)}, \tag{7}$$

where $e$ is the electronic charge, $n_e(r)$ is the local electron density, which may vary in space, $\varepsilon_0$ is the permittivity of the free space and me is the electron mass (MKS units). It should become clear now from the new definition of $N$, that for a particular probing frequency there is a critical density, $n_e(r) = n_c$ above which the probing beam will not propagate through the plasma. We can find out what this critical density is from the above expression, thus:

$$n_c \approx \frac{\omega^2}{80.6}. \tag{8}$$

*(**Task 1**: Try to estimate it yourself. Have you found the same?)*

So now, we can rewrite:

$$N = \sqrt{1 - \frac{n_e(r)}{n_c}}. \tag{9}$$

Due to the presence of the plasma the phase difference, $\Delta\varphi$ , between the working path and the reference path of the interferometer will be

$$\Delta\varphi = \frac{2\pi}{\lambda_0}\int_0^L [1 - N(n_e(r))]\mathrm{d}L. \tag{10}$$

In most practical cases the frequency at which the interferometer is operated will be chosen far higher than the plasma frequency: $n_e \ll n_c$. In this case the binomial expression of $N$ to fist order can be used,

$$N \approx 1 - \frac{1}{2}\frac{n_e(r)}{n_c}, \tag{11}$$

which then gives us quite a neat expression for the phase shift

$$\Delta\varphi = \frac{0.845\times 10^{-6}}{\omega}\int_0^L n_e(r)\mathrm{d}L = \frac{0.845\times 10^{-6}}{\omega}\langle n_e\rangle L \quad \text{[rad]} \tag{12}$$

where $\langle n_e\rangle$ is defined as the density averaged over the beam path through the plasma, 'line-integrated density'.

## 1.3 The Basic Components of an Interferometer

The most important parts that are used for interferometry are the microwave generator, waveguides and detection diode.

The interferometry on the GOLEM tokamak uses a so-called **Gunn diode oscillator** as a microwave generator. The available microwave generator was made in the Ukraine. According to documentation, its output power is 60 mW and frequency is 71 GHz.

Although microwaves (like all electromagnetic radiation) propagate through free space, it is necessary to have some way of guiding them where they are to be used. **Waveguides** are a way of achieving this. A waveguide is generally a hollow pipe (not necessarily circular in cross-section) made of conductive material. The wave traveling inside is confined and no energy is lost by radiation.

The microwave signal is detected by means of a **detection diode** at the GOLEM tokamak.

There is one additional important component installed on the GOLEM tokamak - the electronic circuit that evaluates the phase shift. With an analog evaluation, the results are immediately ready. It should provide density data precise enough for most of the work.

## 1.4 Experiments

Another tasks will be given to you afterwards.

## 1.5 Further Readings

http://golem.fjfi.cvut.cz/wiki/Library/GOLEM/MastThesis/15MatenaLukas.pdf

Hugenholtz, C.A.J., "Microwave interferometer and reflectometer techniques for thermonuclear plasmas", 00/1990, Ph.D. Thesis Technische Univ., Eindhoven (Netherlands)

# 2 Investigation of Plasma Fluctuations by Mean of Probes with Jordan Cavalier: cavalier@ipp.cas.cz

## 2.1 Introduction

In fusion technology, one very important issue is to understand how the plasma particles drift towards the wall, as it caracterizes confinement time and thus machine performances. Usually, this transport of particles is described using the classical diffusion law (so called Fick's law), similarly to what is done in fluids (gas or liquid). Unfortunately, diffusion theory fails to predict correctly plasma transport in tokamaks by order of magnitudes, likely due to non-diffusive processes that we shall name turbulence. Consequently, there are huge efforts in the fusion community to understand turbulent transport in plasmas. To this purpose, more and more sophisticated diagnostics are being developed that should be able to give information in very hot environments, from few electron volts to kilo electron volts[1]. We here focus on the oldest, yet used worldwide, diagnostic called *probe* or *Langmuir probe*.

Invented in the twentieth of last century by an American called Irvin Langmuir, a Langmuir probe consists of a simple conducting wire, usually made of tungsten, placed in contact with the plasma. Since plasma is a mixture of charged particles, one is expecting electrical currents (either electrons or ions) to flow through the probe, from which one could infer important plasma parameters, providing a little bit of theory. To understand this principle in a more comprehensive way, we shall make an analogy with the current flowing through a simple electrical component, a resistor (see Fig. 2a). When changing the voltage difference $U$ at the edges of a resistor having an electrical resistance $R$, the current $i$ flowing through this ideal element follows the well known Ohm's law: $U = Ri$. Thus, there is a trivial way to experimentally measure $R$: for different values of the potential difference $U$, one can record the different values of the current $i$ and obtain Fig. 2b. This curve is called I-V characteristic (I-V stands for current-voltage) and, in the case of the resistor, its slope provides the $R$ value.

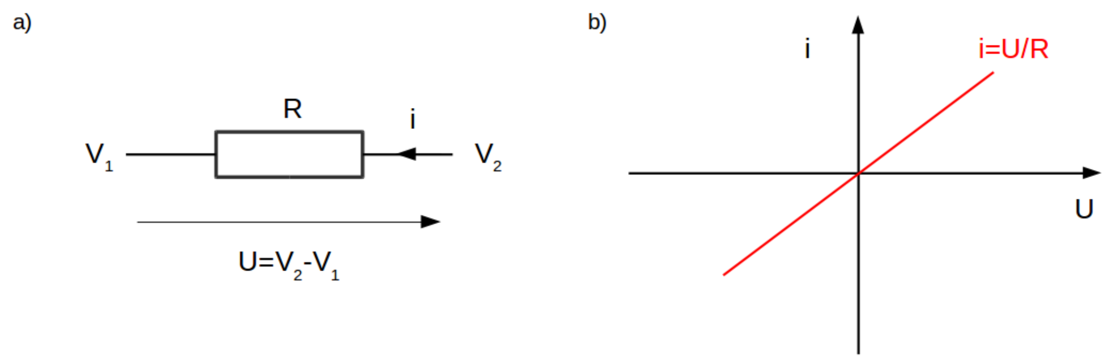


Figure 2: a) Schematic of a resistor. $R$ is the electrical resistance, $i$ the current flowing through the resistor, $V_1$ and $V_2$ the different voltages/potentials applied to the resistor and $U$ is the potential difference. b) I-V (current-voltage) characteristic of a resistor. The slope of the red curve gives information on the electrical resistance $R$.

In plasmas, a similar thing is done with the Langmuir probe. Now, the plasma is the electrical component[2](see Fig. 3a) from which we want to know some quantities, and the Langmuir probe

[1] 1 eV $\simeq$ 11 600 K $\simeq$11 300°C

[2] For those who are passionated by electricity, the plasma can be characterised by a dynamical resistance $R$, inductance $L$ and conductance $C$. The fact that these quantities are dynamical (meaning that they change in time) makes their estimation non obvious.

allows to collect plasma particles and let the current flow from the plasma to the electronic. In the same manner as for the resistor, the current $I_{probe}$[3] is then recorded for different probe potential ($V_{probe}$) with respect to the plasma potential ($V_{plasma}$) and one obtains the I-V characteristic (see a typical I-V characteristic in Fig. 3b). One can notice that the dependence of the probe current $I_{probe}$ on $V_{probe}$ is somewhat more complicated than in the case of the resistor - it is more complicated than $U = Ri$ now. Nevertheless, the knowledge of this curve allows plasma physicists to estimate local quantities as exciting (at least for them!) as the electron plasma density $n_e$, the temperature $T_e$ of the electrons in the plasma and the potential of the plasma $V_p$[4].

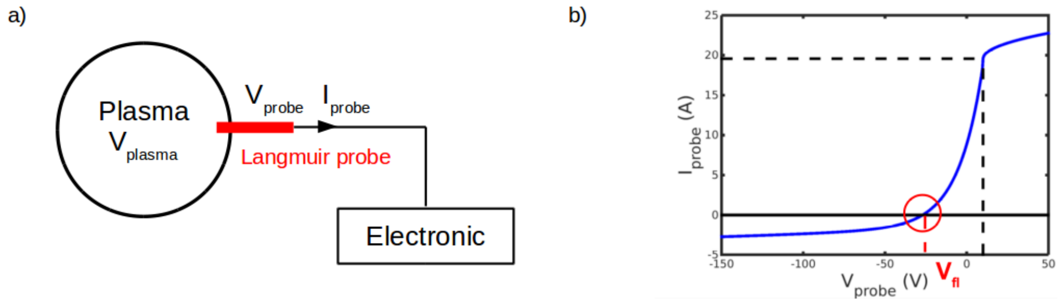


Figure 3: a) Schematic of a Langmuir probe at a potential $V_{probe}$ imposed by the electronic probing the plasma that has an inner potential $V_{plasma}$. b) Typical I-V characteristic that one can obtain by biasing (=impose the potential of a probe) a Langmuir probe. The dashed lines show where the plasma potential $V_{plasma}$ is and the red circle shows the so called floating potential $V_{fl}$.

In this document, we will not describe the whole I-V characteristic response of the plasma as it is beyond the scope of the Golem experiment. However, we will focus on a special potential of the curve, *the floating potential* $V_{fl}$ that is shown by a red circle in Fig. 3b. As one can notice from the figure, this potential is the potential for which there is zero net current flowing from the plasma to the electronic through the probe. In other word, at the floating potential one has:

$$I_{probe}\left(V_{fl}\right) = 0 \tag{13}$$

This condition is in particular achieved when the probe is only in contact with the plasma and isolated from anything else (electronic, ground potential etc...) and that is why this potential is called the floating potential: the plasma and only the plasma determines the potential value, so that the potential has a similar behavior as a boat floating on the sea that goes up and down depending on the waves passing under it. As we shall see, it is quite interesting to measure this potential because it can be related to some physical quantites that scientists need to access to calculate and understand the transport of particles in fusion devices. Since the probe needs to be isolated to float, one just needs to connect to it a voltmeter or its equivalent. The high impedance of these devices[5] is high enough to prevent any significant current to flow through it so that the probe is still floating (condition given by Eq. 13 still satisfied).

[3] This current is typically called probe current because the Langmuir probe allows the current to flow through the electronic.

[4] Important: A potential alone has no physical meaning, it should always be given with respect to a reference one (usually the ground/earth potential, which is taken by digging a wire under a building, for example, and plugging it to the electronic). This is similar to height that should be given with respect to a reference point, usually the sea level. But remember that the sea level also has a certain height with respect to the Earth's core. It all depends on the reference that one chooses. In the rest of this document, we will take the earth potential as a reference.

[5] The standard impedance for voltmeters is 1 MegaOhm, usually much greater than the plasma one.

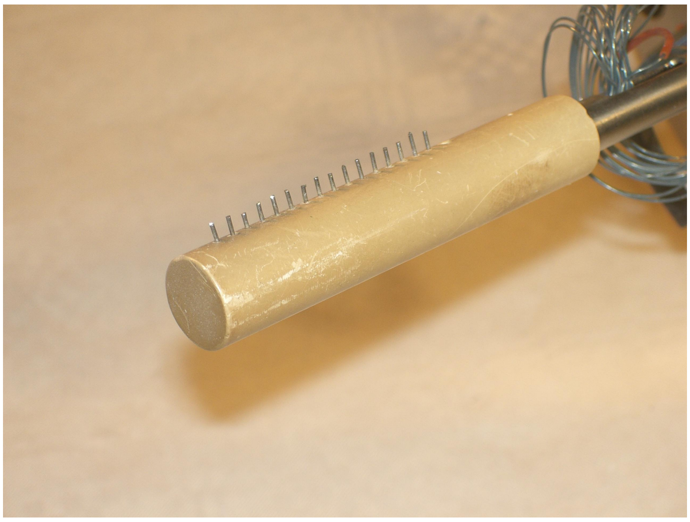

Figure 4: The rake probe on GOLEM. It is made of 16 Langmuir probe pins. Only 12 probes are measuring due to the number of available acquisition channels. These probes are measuring the floating potential at different radial location from the centre of the plasma to the plasma edge. Two consecutive probes are separated by 2.5 mm.

We now state without any demonstration the link between the floating potential and some of the plasma parameters:

$$V_{fl} = V_{plasma} - A \times T_e \tag{14}$$

where $A$ is a constant not exhibited for conciseness.

From Eq. 14, one sees that the floating potential is a linear function of the plasma potential $V_{plasma}$ and the electron temperature $T_e$, so that in principle, the knowledge of one of these quantities and of the floating potential allows to determine the third one. You will see later in the section devoted to your tasks that this formula is quite important to study plasma fluctuations and other plasma properties linked to the transport of particles (remember that understanding particle transport in tokamak is a major issue for the fusion community). Before that, we shall present the setup of the probe used on the machine Golem located in Prague, Czech Republic.

## 2.2 The rake probe on GOLEM

On GOLEM[6], they do not have a simple single Langmuir probe for probing the plasma, no, they have 16 (see Fig. 4)! The position of each probes corresponds to a different radial location inside the vessel and because of the shape of the probe head that looks like a rake used to sweep your garden off tree leaves, it is called *rake probe*. At the moment, they are only 12 acquisition channels available on GOLEM, so only 12 probes are measuring radial profiles of floating potential. The probe at the edge of the yellow ceramic tube is the probe measuring the deepest inside the confine plasma (position r = 70 mm) while the last probe (position r = 85 mm) measures behind some plasma limiter shadow and thus is not in the confined plasma anymore (ask me question directly if you want more infos about this). The distance between two consecutive probes is **2.5 mm**. We will use the signals from these probes to successfuly fulfill your tasks.

[6]Small tokamak located in the beautiful city of Prague, Czech Republic. You will connect remotely to it and by setting online some physical parameters you will create your first plasma discharge in a tokamak! See the presentation on GOLEM for more details.

During the Golem experiment, you will have the possibility to change four parameters that I recall here as we should scan them for our tasks. More details about it would be provided during the presentation about the GOLEM experiment.

- $U_B$ = voltage that controls the toroidal field coils.
- $U_{cd}$= voltage that controls the poloidal field.
- $t_{cd}$= time delay between start of toroidal field and poloidal field.
- $p_{wg}$ = filling pressure of the working gas (Hydrogen or Helium).

## 2.3 Tasks

As already said, we will use the floating potentials $V_{fl}$ of the 12 measuring Langmuir probes of GOLEM together with Eq. 14 for our tasks. This equation is quite important because it allows to infer some physical quantities from measurements of $V_{fl}$. However, since we have only one equation but two unknowns[7] ($T_e$ and $V_{plasma}$), we have to make an assumption on one of them to determine the second. In this work, we are more interested about $V_{plasma}$ as it can be linked to the electric field $E_r$ (see Eq. 16 below) that is an important quantity to estimate the transport of particles towards the wall and thus to estimate machine performances. Consequently, we will do an assumption on the electron temperature $T_e$ in the following.

**Assumption for our tasks:** The electron temperature $T_e$ is not fluctuating = is constant with time.

### 2.3.1 Preliminary work - homework

Before describing what your tasks would be, I will ask you to do a quite simple preliminary work. Since we said that $T_e$ is not fluctuating, I would like you to show using Eq. 14 that the fluctuations of the floating potential $V_{fl}$ (the potential we are measuring with probes) are equal to the fluctuations of the plasma potential $V_{plasma}$. If the assumption is true, it means that the probes provide a direct measurement of the plasma potential and consequently of the local electric field!

So try to show that

$$\text{Eq. 14 and } T_e \text{ not fluctuating } \Rightarrow \tilde{V}_{fl} = \tilde{V}_{plasma} \tag{15}$$

**Hint:** Write each quantities that are fluctuating as $f_{tot}(t) = f_{mean} + \tilde{f}(t)$ in Eq. 14, where $f_{mean}$ is the mean value of $f_{tot}(t)$ and $\tilde{f}(t)$ the fluctuating part.

Do not worry if you do not know how to show that, we will do it together the day of the experiment.

### 2.3.2 First task: optimization of plasma parameters

- **Goal of the task:** Try to find the plasma parameter set ($U_B$, $U_{cd}$ and $t_{cd}$) that minimize plasma fluctuations and thus increase confinement time and machine performances. Do one more experiment with this particular set of parameters and check the level of fluctuations. Conclude.

We said that particle transport to the wall determines machine performances and that it is linked to fluctuations of $V_{plasma}$. In fact, the greater these fluctuations the more transport and thus worse is the confinement and machine performances. In this task, you will be only looking at the signal of

[7] $A$ is also unknown but in fact in the simple probe model it can be estimated knowing the working gas and probe geometry. Here we do not need to estimate it so the formula is not explicitly shown.

one probe, the one that is the deepest into the plasma. Choosing carefully a time window for which plasma conditions are stable (we will look together), you will compare the fluctuation level of $V_{fl}$, and thus $V_{plasma}$ as we assume Eq. 15 to be true, for different values of the parameters $U_B$, $U_{cd}$ and $t_{cd}$. For example, keeping $U_B$ and $U_{cd}$ constant, you will have to find the value of $t_{cd}$ for which the fluctuations $\tilde{V}_{fl}$ are the weakest. Doing the same for $U_B$ and $U_{cd}$, you will determine the set of parameters for which fluctuations should be minimum. At the end, you will do one more experiment for this set of parameters and see if, indeed, fluctuations are smaller. Depending on the result, we will conclude.

### 2.3.3 Second task: estimation of the fluctuating poloidal velocity from probes

- **Goal of the task:** Compare radial profiles of the fluctuating (poloidal) velocity for different plasma parameters ($U_B$, $U_{cd}$ and $t_{cd}$). Understand the evolution of the curve with respect to the radial position. Conclude on the effect of each plasma parameters on the velocity.

Since we assume that Eq. 15 is true, it means that the rake probes are directly measuring the plasma potential fluctuations. One can then link these fluctuations to the fluctuations of the radial electrostatic electric field by writting that

$$\tilde{\mathbf{E}}_\mathbf{r} = -\frac{d\tilde{V}_{plasma}}{dr}\mathbf{u}_\mathbf{r} = -\frac{\tilde{V}_{LP1} - \tilde{V}_{LP2}}{L}\mathbf{u}_\mathbf{r} \tag{16}$$

where $\mathbf{u}_\mathbf{r}$ is a vector oriented from the centre of the machine to the outer edge, L is the distance between two probes (=2.5 mm), $\tilde{V}_{LP1}$ and $\tilde{V}_{LP2}$ are the floating potential of two consecutives Langmuir probes and the symbol ˜ stands for fluctuations. Remember that this equation is only true because we assumed $\tilde{V}_{fl} = \tilde{V}_{plasma}$!

We can then link the fluctuations of the electric field to the fluctuations of the poloidal velocity[8]

$$\tilde{v}_p = \frac{\tilde{E}_r \times B}{B^2} \tag{17}$$

where $B$ is the magnetic field that we assume to be constant.

Using consecutive probes, you will then calculate the radial evolution of $\tilde{v}_p$ and try to understand its behavior. It will require to find a suitable time for which plasma conditions are constant (we will look together). You will then compare the radial profiles for different parameters $U_B$, $U_{cd}$ and $t_{cd}$. For example, keeping $U_B$ and $U_{cd}$ constant, you will look at the evolution of the radial profiles by changing $t_{cd}$. Doing the same for each parameters, you will conclude on the effect of these parameters on the fluctuating velocity.

[8] This velocity is classical in plasma physics. It is called the E cross B velocity and is perpendicular to both the electric and magnetic fields. Since $\mathbf{E}_\mathbf{r}$ is measured in the radial direction and $B$ mainly in the toroidal direction, $v_p$ is in the poloidal direction.

# 3 Plasma Centre Position Reconstruction Using Magnetic Diagnostics with Ondřej Kudláček: onk@ipp.mpg.de

## 3.1 Motivation & Basic principles

One of the main goals of the tokamak development is to reduce the contact of the hot plasma and the wall of the vacuum vessel as much as possible. This task is not easy as the plasma moves in the vessel due to many fluctuations and instabilities. For example, a minor improvement of plasma confinement leads to the shift of the plasma column outside the vessel. If this shift is not compensated, the plasma touches the wall, cools down and the good properties of the plasma are lost. The processes leading to plasma shifts are neither predictable, nor avoidable. Therefore every good tokamak must have a system that observes what is happening with the plasma in real time and a system of actuators capable to react on changes inside the plasma.

Let us explain basic principle of the actuator systems on the example shown in Fig. 5. In this case, suppose that we want to move the plasma outwards from the axis of the device (for example because the plasma touches the inner wall). The actuator we will use for that are the poloidal field coils, that will generate required magnetic field. If we want to move the plasma outwards, we need to generate current in the poloidal field coils such that the plasma is attracted outside and repelled from inside. As the force between parallel currents is attractive, we generate current that has the same direction as the plasma current in the outer poloidal field coils. On the contrary, we generate currents in the *opposite* direction with respect to the plasma current in the inner coils to repel the plasma outwards. The magnitude of the generated current is very dependent on the control system, the analysis in this paragraph offers just the basic picture.

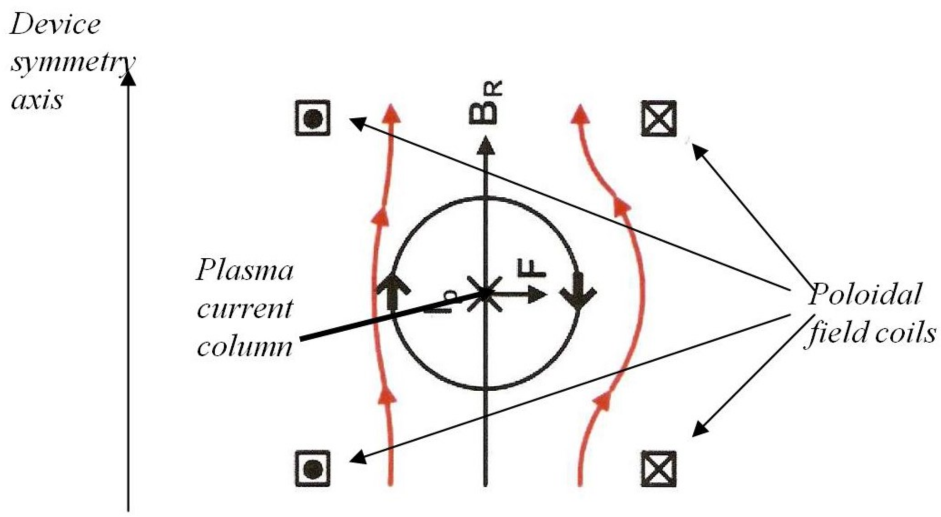


Figure 5: An example of moving plasma current column outside the tokamak vessel.

As you might have noticed, there is one important question not answered above: how do we measure the plasma position? The method has to be fast enough ( 1 computation in less than 0.1 ms), sufficiently precise, and reliable. The most common method is the measurement by magnetic diagnostics. Designing and testing of a measurement algorithm for GOLEM will be your task. First of all, let us describe the basic principles of the magnetic diagnostics.

### 3.2 Principles of Relevant Magnetic Diagnostics

The magnetic diagnostics we will use in this task is based on measurement of the magnetic field generated by the plasma. Each sensor consists of a coil (usually referred as Mirnov coil or pick-up coil) and a measurement of the voltage induced on the coil. The induced voltage $U$

$$U = -C\frac{\mathrm{d}B}{\mathrm{d}t}, \tag{18}$$

where $B$ is the projection of the magnetic field to the direction perpendicular to the coil loops, $t$ is the time and $C$ is a constant representing the coil geometry (area, number of loops). Due to strict requirements on the precision of $C$, this value is usually determined experimentally (remember that we can not include random, but unavoidable, manufacturing error to our computations).

However, for the measurement of the plasma current centroid position the magnetic field $B$ is relevant, not its derivative that is directly measured. Therefore we need to integrate the equation 18. This can be done by two means: either numerically, or by an analog integrator. The later method is preferred due to better precision.

Another issue arises from imperfect grounding: the measurement of the voltage at the magnetic sensor is not equal to 0 once there is no external magnetic field and this irrelevant voltage $u_{par}$ is integrated! This can (and mostly does) generate a big error in the magnetic field computation. Therefore this effect must to be subtracted as follows:

1. Integrate the voltage measured on the sensor, the integration must start BEFORE the plasma discharge.
2. The voltage $u_{par}$ is usually constant (if not, you have a problem that is pretty hard to solve) and you can estimate the value of $u_{par}$ from the tangent of the integrated voltage before the discharge is started.
3. You subtract from your integrated voltage $U$ the parasitic part $u_{par} \cdot t$ to get the real signal generated by the tokamak magnetic fields (not just plasma!).

After applying this procedure, you have the required signals and can start analyzing the plasma position.

### 3.3 Magnetic Diagnostics on GOLEM & Your Tasks

The scheme of available Mirnov coils on GOLEM is shown in Fig. 6. All the Mirnov coils are calibrated, the calibration constants will be obtained from the GOLEM team during the experiments. From these four Mirnov coils, we will try to deduce the plasma current centroid position by the following means:

1. Approximating plasma by a thin straight conductor and assuming that the displacements are small compared to GOLEM minor radius, you will derive a simple formula for plasma centre position computation. Afterwards, you will test whether the formula meets theoretical expectations while moving the plasma in radial/vertical direction using pre-defined waveform in external poloidal field coils.
2. We will also consider the plasma toroidal geometry. Using Biot-Savart law, we will compute expected magnetic field for various plasma positions in the vessel. Afterwards, we will derive the plasma position from the experimental measurements using convenient method. Again, we will compare the results of this method with theoretical expectations and with the method derived in the first tasks.

3. If you find the above mentioned tasks too easy and boring, we will **try** to find a convenient representation describing the effect of poloidal field coils and conductive structures in Golem on the magnetic field measured by the Mirnov coils. This could improve the above mentioned algorithms, assuming that the measured magnetic field comes just from the plasma. Please keep in mind that this is a difficult task and contact me in advance if you are interested!

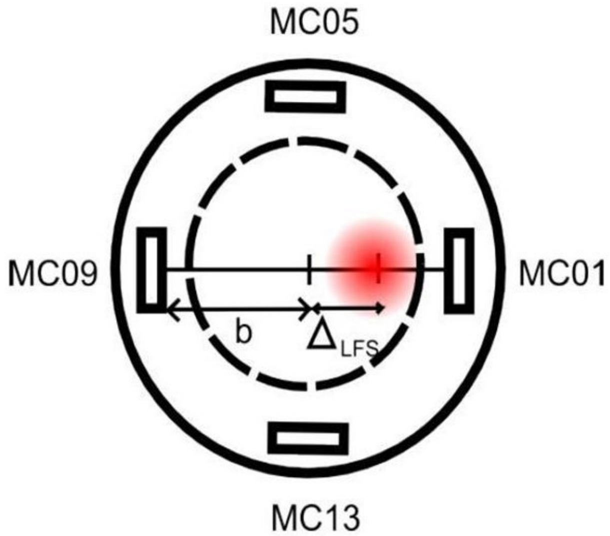


Figure 6: Location of the Mirnov coils with misplaced plasma column represented by red circle. The black dashed line represents limiter.

# 4 Safety Factor - a Signature of Plasma Stability in Tokamaks with Branka Vanovac: v86bra@gmail.com

## 4.1 Safety Factor

Tokamak is a toroidal device where the particles are confined by the presence of strong toroidal magnetic field generated by the external coils, and poloidal field generated by the plasma current (Fusion Machines lecture). Thus, a resulting field lines follow the helix (Fig. 7) around the torus making a toroidal and poloidal turns simultaneously. Ratio of number of toroidal turns $m$ and number of poloidal turns $n$ that the field line makes before closing up on itself is named safety factor $q = m/n$.

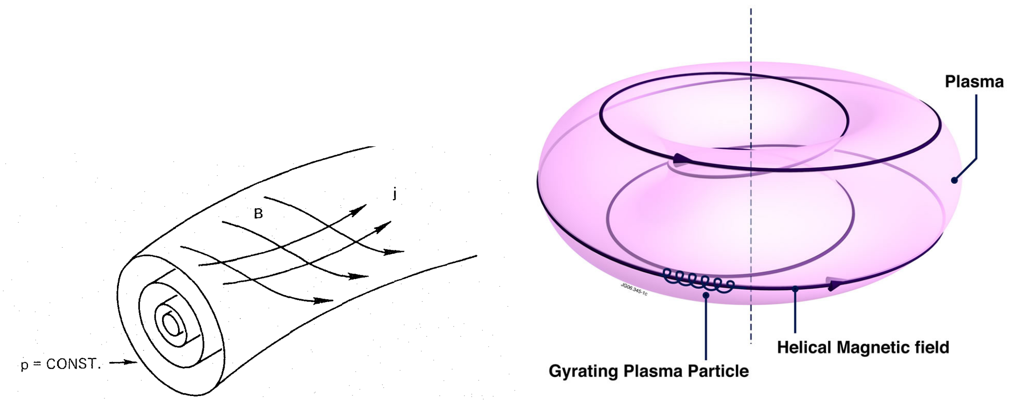


Figure 7: Left: Flux surfaces along which plasma pressure, temperature and the density are constant. Magnetic field lines $\vec{B}$, current density $\vec{j}$. Right: m/n = 2/1 corresponding to a q = 2 flux surface.

In terms of plasma quantities safety factor at the plasma edge is defined as:

$$q(a) = \frac{aB_t}{RB_p} \tag{19}$$

with the poloidal field defined as:

$$B_p = \frac{\mu_o I_p}{2\pi a^2} \tag{20}$$

where $a$ is a minor radius of the plasma column, $R$ is the major radius; $B_p$ poloidal magnetic field; $B_t$ toroidal magnetic field. Using the numbers for the minor and major radius of GOLEM, the edge safety factor can be expressed as:

$$q(a) = 90.3\frac{B_t}{I_p}[T, kA] \tag{21}$$

where $B_t$ has units of [T], and $I_p$ of [kA]. You should be able to obtain the same number :)! For instance $q = 1$ means that the field line will do one toroidal and one poloidal turn, whereas a $q = 2$ field lines has to go twice toroidally and once poloidally. A typical $q$ profile is given in the Figure 8, and it monotonically increases from the plasma center ($r/a = 0$) towards the plasma edge ($r/a = 1$). Thew word 'safety' here depicts the MHD[9] stability of the plasma. The higher the values of the edge safety factor, the more stable plasma is.

[9]Magnetodydrodynamics: Plasma is treated as a electrically charged conductive fluid with dominant electromagnetic forces described by Maxwell equations

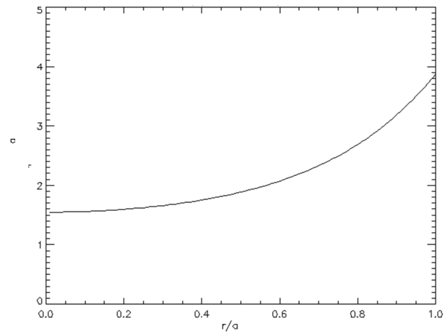


Figure 8: $q$ profile as a function of normalized minor radius $r/a$.

### 4.2 Plasma Instabilities in GOLEM Tokamak

We have mentioned the stability of the plasma. What does it mean, we will see in following: As seen in the Figure 7, magnetic field lines are forming flux surfaces $q$, and in the equilibrium state plasma pressure and the temperature are constant along them. So, the plasma equilibrium is always an interplay between the configuration of the magnetic field lines, plasma pressure (that acts outwards) and the $\vec{j} \times \vec{B}$ force (acts inwards). Also the vessel wall and the plasma shape plays an important role in the plasma stability. Plasma, for example, can be elongated using the external field coils, and therefore increase the stability. So, there are many parameters that are coming into the game when it comes to determining the plasma stability condition and for the further reading I would suggest you to check the references listed in the Section 4.4. However, for the scope of this exercise, it is important to know that all the instabilities (perturbations, modes) in the tokamak are appearing as a periodic oscillations which can be Fourier decomposed in the toroidal and poloidal direction as:

$$\xi(r,\theta,\phi) = \xi(r)e^{i(m\theta - \frac{n}{R_O}\phi)} \tag{22}$$

where $\theta$ and $\phi$ are toroidal and poloidal angle, $m$ and $n$ are respective mode numbers. Yes, those are the same mode numbers that were defining the edge safety factor. Again the stability analysis has shown that this modes will appear on the resonant flux surfaces, and those are the surfaces with the integer $q$ factor. So, looking in the figure 8, one can note that instabilities can occur at the $q = 2$ and $q = 3$. By knowing the mode number of the instability and the $q$ profile of the plasma, one can easily localise (determine the position) where the mode is 'sitting'. Now if we think of the modes developed on the flux surfaces somewhere inside the plasma, those modes can propagate along the surfaces in form of waves. So they can be characterise by means of the wave characteristics (frequency, wave number, wavelength), they obey constructive, destructive interference. They can superimpose appearing in a form of higher or lower mode numbers. And so, those instabilities are rotating together with the plasma,and their presence can be captured by the magnetic pick up coils - Mirnov coils. Brief explanation of the Mirnov coils principle is given below [10]:

#### 4.2.1 Mirnov Coils

Rotating structures will cause the perturbation in the Mirnov coils signals, therefore analysing the signal picked up from the Mirnov coils , we can follow the temporal evolution of the rotation of detected instabilities. The poloidal distribution of Mirnov coils around the plasma column is shown

[10] You should also check the Task 3 of Ondřej Kudláček

in the Figure 9. Determination of the mode numbers is out of the scope of this exercise, but however I

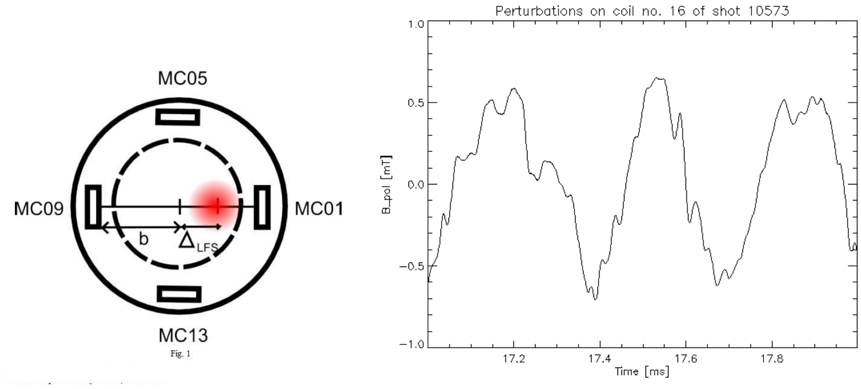


Figure 9: Poloidally distributed Mirnov coils on GOLEM tokamak and typical perturbation signal detected by the coils. Source: `http://golem.fjfi.cvut.cz/wiki/`

will only give you the hint that with poloidal distribution of the coils every coil will pick up the signal for the poloidally rotating structure. By means of cross correlation of these signals and representing it in dependence on the poloidal angle, one can find the poloidal mode numbers $n$. Knowing that those are appearing on the integer $q$ values, it should not take long to be able to figure out where they are sitting, at which position of the plasma radius. However, even if it sounds easy, this exercise can get very complicated. You can also imagine that advanced machines nowadays have toroidally distributed coils that can help us determine the toroidal mode numbers.

## 4.3 Tasks

So far we have seen that edge safety factor depend on plasma current and toroidal magnetic field. Variation of these parameters will give us different values of the edge safety factor. From there on we will try to identify the presence of modes detected by Mirnov coils, and see the frequency range of their rotation.

### 4.3.1 Task 1

So, as for the little bit of calculus gymnastics lets derive the final expression for the edge safety factor. Starting from the Equation 19 derive the final expression for the edge safety factor assuming a cylindrical geometry .

### 4.3.2 Task 2

Show the time traces of all the relevant parameters necessary for determination of the $q(a)$. Calculate edge safety factor for all available discharges and select the discharges of the highest, moderate and lowest edge safety factor.

### 4.3.3 Task 3

Now since we have obtained the edge safety factor for all the discharges and determined the extreme values, for those discharges using fast Fourier transform (FFT) obtain the spectrograms of the signals fro Mirnov coils. Compare the frequency range of the modes with the values of the edge safety factor.

## 4.4 Further Readings

Boyd-Sanderson, *The Physics of Plasmas*
Jeffrey Freidberg, *Plasma Physics and Fusion Energy*
J.Z.Wesson, *Tokamaks*

# 5 Estimation of Energy and Number of Runway Electrons with Ondřej Ficker: ficker@ipp.cas.cz

Runaway electrons are high energy particles that appear in tokamaks under certain conditions - e.g. in case of low plasma density. These particles are highly undesirable during normal operation as they can damage important components of the device. Therefore it is of crucial importance to study the generation and losses of these particles in plasmas of smaller devices and use the results to secure safe operation of ITER and future large reactors. During the tasks described below you will become familiar with the conditions suitable for generation of these fast particles in tokamak GOLEM and you will use one of the most common detectors (Scintillation detector) to study how many of the runaway particles are generated during the plasma discharge.

## 5.1 What are Runaway Electrons?

As was mentioned above, runaway electrons are just the electrons in a plasma that have reached high velocities and may be further accelerated by the electric field that is naturally present in the tokamak.

### 5.1.1 Electric Field in the Tokamak

As you probably already know, a significant part of the tokamak principle is similar to the principle of an electrical transformer, thus there is the toroidal electric field that drives the plasma current. The field is necessary for the tokamak to work but on the other hand it can cause problems. The acceleration in the electric field has to be balanced by particle collisions for all particles in order to have a thermal plasma. If it is not balanced, then runaway electrons appear.

### 5.1.2 Coulomb Drag Force and Particle Velocity

To take a closer look at the Coulomb friction force (due to collisions), see Figure 10 - the 'drag function' peaks at the thermal velocity and drops quickly as $v^{-2}$ for the faster electrons, the electric accelerating force is independent on the velocity, therefore it can be represented as a line parallel to the velocity axis. What are the relations of these two curves to the basic plasma parameters and discharge characteristics? Well, the magnitude of the electric field $E$ moves the horizontal line up and down, the electron density $n_e$ of the plasma scales the curve and the electron temperature $T_e$ moves the peak along the velocity axis. These three parameters are the most important and we can define some critical values using them:

- The Dreicer field as the field in which all electrons run away

$$E_D = \frac{n_e e^3}{4\pi\varepsilon_0^2 k T_e} \ln\Lambda \tag{23}$$

- The critical field as the minimal electric field required for RE to appear

$$E_c = \frac{n e^3}{4\pi\varepsilon_0^2 m_e c^2} \ln\Lambda. \tag{24}$$

- The critical velocity as the minimal velocity required for the electron to become a runaway electron

$$v_c = \sqrt{\frac{n e^3}{4\pi\varepsilon_0^2 m_e E} \ln\Lambda}. \tag{25}$$

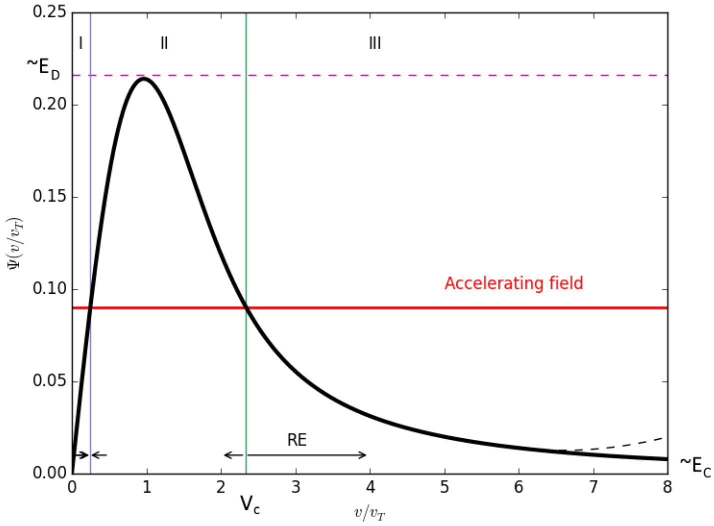


Figure 10: The dependence of the Coulomb friction force on the velocity. The runaway region is on the right.

In all relations $e$, $k$, $c$, $\varepsilon$ and $m_e$ are well known constants and $ln\Lambda$ is the Coulomb logarithm - very weak function of plasma parameters usually close to 15 in fusion plasmas.

## 5.2 Detection Method

It is very difficult - and in GOLEM almost impossible - to detect a small number of runaway electrons in the plasma, thus the best option is to focus on their interaction with atoms of limiters and the vessel in general (producing bremsstrahlung radiation and line emission). Every particle with energy in the order of hundreds of keV creates photons of very high energy (hard X-ray, but the energy may be deep in the gamma region, it is not called gamma radiation just because it is not created in the nucleus) when it hits a high density material. Using this knowledge we can easily measure whether a significant number of runaway electrons was created. All runaways hit the wall sooner or later due to drifts or magnetic field imperfections and perturbations. To detect the HXR radiation we use a scintillation detector based on the NaI(Tl) crystal and classic high voltage photo-multiplier (PMT).

### 5.2.1 Principles of Scintillation Detector

In the scintillation crystal the HXR photon is transformed to the shower of low energy photons (usually in the visible range) as it excites the atoms of the material. The shower of photons is then transformed in the photocathode to the shower of electrons which is amplified on the cascade of dynodes to create a measurable current/voltage signal. The height of the voltage peak is directly proportional to the incident photon energy. Our PMT is in the current (continuous) regime.

### 5.2.2 Calibration of the Detector

The calibration of the HXR scintillation detector in energy was done with the help of gamma radiating isotopes $^{60}Co$ and $^{137}Cs$. The gamma quanta generated by the nuclear transitions correspond to energies 1.1732 MeV and 1.3325 MeV for $^{60}Co$ and 662 keV for $^{137}Cs$.

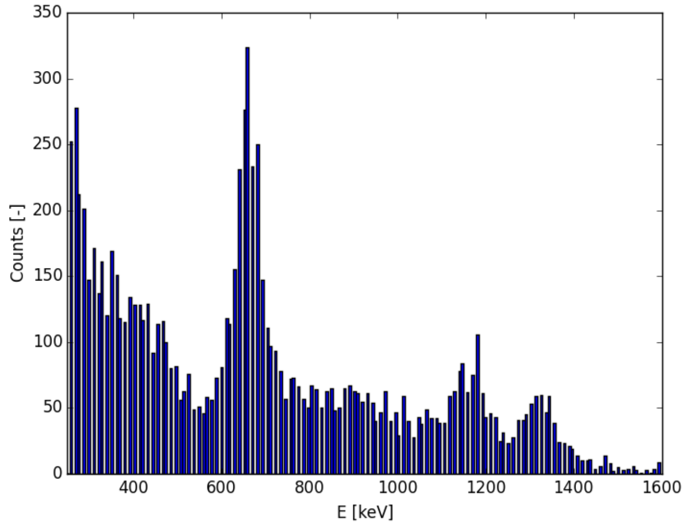


Figure 11: Spectrum of $^{137}Cs$ + $^{60}Co$ radioactive sources measured via NaI(Tl) scintillation detector used at GOLEM.

The three full absorption peaks are obvious in the spectrum in Figure 11. According to the voltage of the power source that we will use, the voltage signal of the $^{137}Cs$ full absorption peak will be

$$U_{Cs}[V] = \exp(0.011 * (U_{PS}[V] - 789.4)). \tag{26}$$

This can give us an estimate of the HXR (RE) energy. Details will be specified during the experiments.

## 5.3 Tasks

### 5.3.1 A bit of Theory: Acceleration of Electrons in the Vacuum and How Many of Them May Become Runaways in a Maxwellian Plasma?

a) First task will be a little bit theoretical. Try to estimate the energy that would **electrons achieve in a betatron** (electron accelerator with stable electron orbits) with the **loop voltage values of typical GOLEM discharge**, e.g. 8 V on the time scale of 10 ms. The electric field is in the first approach the loop voltage divided by the length of the magentic axis

$$E = \frac{U_{loop}}{2\pi R_0} \tag{27}$$

In fact very low density tokamak operation is not far from the operation of an electron accelerator, however the acceleration of runaway electrons is still slower compared to better vacuum conditions of the betatron. Express the energy in various units (eV, Joules). Once finished you can apply your formula or integrator on the loop voltage data from the discharges of the task 2. Assume that the RE are **created at the moment of breakdown**.
**Hint**: Don't forget about Albert!

b) Having a knowledge of the three important parameters ($n_e$, $T_e$, $E_\phi$) and assuming nice Maxwellian plasma (we are very theoretical right now :)) try to estimate the fraction of runaway electrons in the plasma ($n_{RE}/n_e$) and the current they can carry using a simple relation $j_{RE} = en_{RE}c$. Derive the general formula first, then use typical GOLEM numbers.
**Hint**: Remember critical velocity as the limit.

### 5.3.2 Lets Run a Density Scan and Detect some Runaways

One of the parameters that are **easy to steer on Golem** is the **pressure of the working gas** in the chamber before the discharge. Given good vacuum conditions, this should be directly proportional to the electron density $n_e$ which is in turn the important parameter affecting runaway electron generation. Therefore we will do a scan in this parameter, 5 discharges with pressures 3,5,10,15,20 mPa would be optimal. The values of the capacitor bank voltage for toroidal magnetic field and Current drive and the timing will be fixed. For each discharge calculate average value of $E_c$ and analyse the signal of the HXR detector, especially these parameters: **time** $t_{start}$ when the first HXR peaks appear after the breakdown, **size of several clear single peaks** $I_{HXR}$ (hopefully we can find some of them), **number of peaks** $N$ and the value of the **integrated signal** over the whole discharge $S$. For the first two quantities you can use standard MATLAB plot with zooming, the third one can be obtained using the *findpeaks* function, and the last one is just a sum of the signal values. Present the results in a suitable form (graphs, tables).

### 5.3.3 BONUS TASK - Try to Correctly Compare the Simple Theory and Measured Values

Try to compare the measured data with the theoretically calculated values, i.e. the energy of the photon from the peak height using Cs calibration data compared to the energy of a electron accelerated in the vacuum and given field. In case of an isotropic spatial distribution of the HXR photons and assuming one electron = one HXR photon, use the knowledge of the detector position (will be specified) and $N$ to estimate the overall number of RE lost to the limiter. How does this cope with the RE fraction of the Maxwellian plasma derived in 1b)? For this you will need also the volume of the GOLEM plasma. Given the mass of the scintillation crystal calculate the dose absorbed by the crystal during one discharge (use $S$ and the transformation to deposited energy that will be supplied).