Hands-on project : Experiment on GOLEM

R. Guirlet on behalf of the French lecturers

Preamble

This document was initially composed for students having some experience in basic tokamak physics and experiments. It aimed at helping them to relate their academic lectures to a real tokamak environment and particularly the GOLEM environment. It has been adapted to suit better the participants of the ASPNF School in Bangkok.

A short description of GOLEM is given in part 1 and a list of questions is given in part 2. You will see that answers are given directly below the questions. Nevertheless, you will learn much by thinking of the question before looking at the answer! For those who did not have a specific course on tokamak physics, the vocabulary and the ideas may be new. We will have some time during the School for group discussions on this document and on GOLEM experiments.

Although we took some care preparing this document, you may find mistakes or typos. You are welcome to indicate them to the coordinators of your discussion group.

1. What is GOLEM?

GOLEM (http://golem.fjfi.cvut.cz/) is one of the very first tokamaks and the oldest tokamak in operation in the world. It started its carreer as TM1 at the Kurchatov Institute in Moscow in the early 60's. It was moved to the Prague Institute of Plasma Physics in 1977, where it was operated under the name of CASTOR until 2006. It was then moved again to the Czech Technical University (CTU) in Prague where it was renamed GOLEM, a parabolic reference to a legend about a powerful creature made by a rabbine in Prague in order to serve and protect the Jewish community (https://en.wikipedia.org/wiki/Golem).

GOLEM was installed, commissioned and is continuously upgraded by Vojtech Svoboda with the aim of training students and young physicists interested in thermonuclear fusion research. This is done both by allowing CTU students to develop new systems or diagnostics for GOLEM and by organising remote experiments with groups in various places around the world [V. Svoboda et al., Fusion Eng. Design 86 (2011) 1310-1314].

1.1. Characteristics of the tokamak

The main parameters of GOLEM are listed in the table below:

Plasma major radius	40 cm
Plasma minor radius	8.5 cm
Max. toroidal field	0.8 T
Max. plasma current	10 kA
Typical plasma duration	15 ms
Working gas	H_2

The plasma cross section is circular.

The vacuum vessel is made of stainless steel. It is usually baked with a series of cycles at 200°C before an experiment and is operated at room temperature.

As an example of the recent developments, the machine has been equipped with a high temperature superconducting poloidal coil, which is still in test.

With the provision that a responsible officer (namely Dr. Vojtech Svoboda) be in the tokamak surroundings for reliability and safety reasons, operation of the tokamak can be performed entirely remotely. This can be done either via a web interface or by secured access to the local linux server which controls the machine. The high repetition rate allows to perform a discharge every 2-3 mn.

1.2. Adjustable parameters

The hydrogen pressure in the vessel is monitored with the help of a pressure gauge. The other parameters which can be adjusted are:

- the toroidal field on the axis B_{Tor};
- the electric field at the breakdown E_{BD};
- the electric field during the discharge E_{CD};
- and (sometimes) the vertical magnetic field B_{ST} which allows horizontal stabilisation of the plasma.

Each of these quantities is controlled through a capacitor bank supplied with an adjustable voltage (denoted U_{Tor}, U_{BD}, U_{CD} and U_{ST} resp.).

In addition to these physical quantities, it is possible to set a delay between U_{BD}, U_{CD}, and U_{ST} (i.e. E_{BD}, E_{CD}, and B_{ST}) and U_{Tor} (i.e. the toroidal magnetic field) onset.

1.3. Diagnostics and measurements

• GOLEM is equipped with a set of coils for magnetic measurements:
- a coil around the transformer core for the loop voltage (U_{loop}) measurement;
- a Rogowski coil around the vessel for the total current measurement $I_{tot} = I_P + I_{chamber}$;
- a flux loop around the vessel in a poloidal section for the toroidal field measurement;
- 4 Mirnov coils in a poloidal section inside the vessel for local magnetic field measurements.

• A photodiode viewing a poloidal slice of the plasma through a midplane port window measures (in relative units) the visible radiation intensity.

• A fast camera can be mounted behind a window for imaging of a poloidal slice of the plasma.

• A set of 20 aligned AXUV detectors (bolometers) for measurements of the radiated power profile.

• Note that the diagnostics are maintained by students who work on them only pat time. As a consequence, the only measurements which are always available are those of the magnetics.

The measurements are stored in a database. A pulse summary with the main plasma parameters is displayed on the experiment webpage. The data can also be retrieved as files for further analysis.

2. How to determine the main plasma physical quantities from the measurements?

2.1. Total current

It can be deduced from the Rogowski coil measurement. The coil measures a permanent (i.e. independent of time) voltage U_{offset}, which is called an offset. It corresponds to the coil bias and has no relation with the discharge. During the discharge, there is an additional voltage $U_{VP}(t)$ related to the current flowing in the vessel and the plasma:

$$U_i^R(t) = U_{offset} + U_{VP}(t)$$

where $U_i^R(t)$ is the total voltage measured by the Rogowski coil. Thus the meaningful voltage (i.e. the part related to the current in the vessel and the plasma) is obtained by subtracting the offset from the measurement obtained during the discharge:

$$U_{VP}(t) = U_i^R(t) - U_{offset}$$

Then, as seen in the lecture on diagnostics, the quantity $U_{VP}(t)$ is proportional to the time derivative of the current flowing across the coil:

$$U_{VP}(t) \propto \frac{dI_{total}(t)}{dt}$$

The linearity coefficient is called a calibration factor. We will write it C_I :

$$U_{VP}(t) = C_I \frac{dI_{total}(t)}{dt}$$

For the GOLEM Rogowski coil, the calibration factor is known: $C_I = 2\times10^{-4}$.

To determine the total current $I_{total}(t)$, we just have to perform an integration over time:

$$I_{total}(t) = \frac{1}{C_I}\int_0^t U_{VP}(t')dt' = \frac{1}{C_I}\int_0^t \left(U_i^R(t) - U_{offset}\right)dt'$$

This is a theoretical formula: in reality, the measurement is not continuous. The system performs a series of measurements separated by a small time interval Δt .

→ How do we have to adapt the theoretical formula to obtain the current?

In practice, to determine the total current, we replace the integral by a sum, assuming that the current does not change during narrow time intervals:

$$\begin{aligned} I_{total}(t) &= \frac{1}{C_I}\int_0^t \left(U_i^R(t') - U_{offset}\right)dt' \\ &\approx \frac{1}{C_I}\sum_{j=0}^{t/\Delta t}\left(U_i^R(t_j) - U_{offset}\right)\Delta t \\ &\approx \frac{1}{C_I}\left(\sum_{j=0}^{t/\Delta t} U_i^R(t_j)\Delta t\right) - U_{offset}t \end{aligned}$$

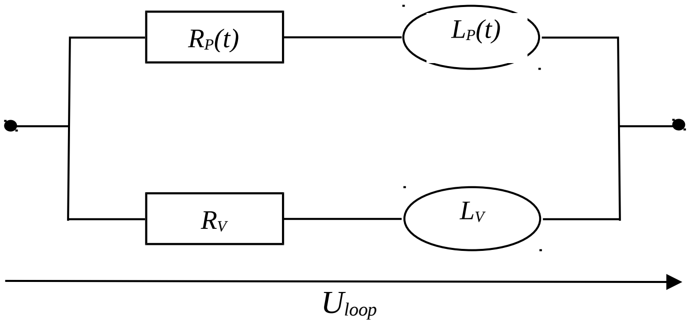
→ How to determine U_{offset} ?

The offset is the bias of the coil before the plasma starts, which corresponds approximately to the first 450 measurement points:

$$U_{offset} = \frac{\sum_{i=1}^{450} U_i^R}{450}$$

2.2. Plasma current

Due to the fact that the vessel is metallic, the current induced in the tranformer after the breakdown flows both through the plasma and through the vessel. The system can be seen as the following electrical circuit:



where the plasma and the vessel are in parallel. On this sketch, the upper branch represents the plasma and the lower branch represents the vessel. $R_P(t)$ and R_V are the plasma and the vessel resistivities and $L_P(t)$ and L_V are the plasma and the vessel inductances respectively. R_V and L_V are characteristic of the vessel and thus independent of time. Note that the loop voltage $U_{loop}(t)$ is measured directly.

→ Write down the circuit equations i.e. the expressions of $U_{loop}(t)$ and $I_{total}(t)$.

The circuit equations are:

$$U_{loop}(t) = R_V I_V(t) + L_V \frac{dI_V}{dt} = R_P(t) I_P(t) + L_P(t) \frac{dI_P}{dt}(t) \quad .$$
$$I_{tot}(t) = I_V(t) + I_P(t)$$

→ How can the plasma current be deduced from the total current and the other available quantities?

It is possible to perform discharges without plasma (e.g. with no gas injected into the vessel). In such discharges, $I_{tot} = I_V$. Therefore, I_{tot} and U_{loop} being known, R_V and L_V can be determined.

In subsequent discharges with plasma, I_V can then be deduced from U_{loop} and subtracted from I_{tot} to obtain I_P.

→ Using Ohm's law (https://en.wikipedia.org/wiki/Ohm%27s_law) applied only to the plasma branch, you will also determine the plasma resistivity.

Once I_P is known, the plasma resistivity $R_P \times 2\pi R_0$ (R_0 being the major radius) can be obtained using the relation:

$$R_P(t) = \frac{U_{loop}(t)}{I_P(t)}$$

2.3. Toroidal magnetic field

→ Using the same principle as for the total current and replacing U_i^R with the appropriate voltage U_B and C_I with $C_B = 1/170$, you will determine the toroidal magnetic field.

As the Rogowski coil, the flux coil measures a permanent (i.e. independent of time) offset voltage $U_{B,offset}$ *. It corresponds to the coil bias and has no relation with the discharge. During the discharge, there is an additional voltage* $U_B(t)$ *related to the magnetic field in the vessel and the plasma:*

$$U_B(t) = U_{B,offset} + U_{B,VP}(t)$$

where $U_B(t)$ *is the total voltage measured by the flux coil. Thus the meaningful voltage (i.e. the part related to the magnetic field in the vessel and the plasma) is obtained by subtracting the offset from the measurement obtained during the discharge:*

$$U_{B,VP}(t) = U_B(t) - U_{B,offset}$$

The flux loop around the vessel gives a measurement of the time derivative of the magnetic field:

$$U_{B,VP} = C_B \frac{dB(t)}{dt}$$

The magnetic field can be obtained by integrating this expression over time:

$$B(t) = \frac{1}{C_B}\int_0^t U_{B,VP}(t')dt'$$

In practice, as the Rogowski coil, the system performs a series of measurements separated by a small time interval Δt *. The practical expression is thus:*

$$\begin{aligned} B(t) &= \frac{1}{C_B}\int_0^t \left(U_B(t') - U_{B,offset}\right)dt' \\ &\approx \frac{1}{C_B}\sum_{j=0}^{t/\Delta t}\left(U_B(t_j) - U_{B,offset}\right)\Delta t \\ &\approx \frac{1}{C_B}\left(\sum_{j=0}^{t/\Delta t} U_B(t_j)\Delta t\right) - U_{B,offset}\,t \end{aligned}$$

2.4. Injected power

In GOLEM, the only power injected to the plasma is by Joule effect (https://en.wikipedia.org/wiki/Joule_heating): as in every electrical conductor, the plasma current and loop voltage are partly converted into heat. Do you remember P = RI^2 = UI?

→ Using this formula and adapting it to the present situation, deduce the injected power from the physical quantities determined above.

$$P_{inj}(t) = P_\Omega(t) = U_{loop}(t) I_P(t) = R_P(t) I_P(t)^2$$

2.5. Electron temperature

The total plasma current is the integral of the plasma current density *j* over a poloidal section of the plasma:

$$I_P = \int_{Section} j.dS$$

$$j_{//} = \sigma_{//} E_{ind}$$

where $\sigma_{//}$ is the plasma parallel conductivity (meaning the component parallel to the magnetic field) and E_{ind} the electric field component in the plasma current direction.

By replacing this expression of $j_{//}$ in the expression of I_P, we find that the plasma current can also be written in the following way:

$$I_P = \int_0^a \sigma_{//} E_{ind} .2\pi r.dr$$

An expression of the parallel conductivity can be found in [J. Wesson, Tokamaks, Oxford Science Publications, 3rd edition (2004), Section 2.16], from which we deduce:

$$I_P = 1.13 \times 10^3 \times \frac{U_{loop}}{2\pi R_0} \frac{1}{Z_{eff}} \int_0^a T_e(r)^{3/2} 2\pi r.dr$$

where I_P is in A, U_{loop} in V, T_e in eV and the induced electric field has been expressed as a function of the loop voltage (note that, due to the lack of information about the local electric field, we assume here a uniform electric field).

The temperature profile $T_e(r)$ is not measured in GOLEM. We will assume a polynomial form:

$$T_e(r) = T_{e,0}\left(1 - \frac{r^2}{a^2}\right)^2 .$$

The only quantity which is not measured is the central temperature $T_{e,0}$.

→ Using the expressions of I_P and $T_e(r)$, you will determine the central temperature (in eV) as a function of the measured quantities (in SI units).

The integral can be calculated analytically and we obtain the central temperature (in eV) as a function of the measured quantities (in SI units):

$$T_{e,0} = \left(\frac{8}{1.13 \times 10^3} \frac{R_0}{a^2} Z_{eff} \frac{I_P}{U_{loop}} \right)^{2/3} .$$

2.6. Electron density

As the diagnostic for density measurements (interferometer) is not always available and reliable, it can be useful to have another method to determine the plasma density.

We will assume that the gas injected in the vessel prior to the discharge is not adsorbed in the vessel wall, and that it is completely ionised (this can be justified a posteriori by the high value of the central temperature compared with the H ionisation potential). In addition, we assume that the plasma is an ideal gas, so that the plasma ion (or electron) density is the same as the injected gas density.

→ Write the ideal gas law (https://en.wikipedia.org/wiki/Ideal_gas_law) and explain the physical quantities:

$$PV = Nk_BT$$

where P is the pressure, V is the volume of gas, N the corresponding number of molecules, k_B *is the Boltzmann constant and T is the gas temperature.*

→ Determine the electron density using the appropriate assumptions.

Notice that N/V is the average molecule density. In GOLEM we work with hydrogen (H_2). If we assume that all molecules are dissociated and ionised, each molecule will contribute 2 electrons to the total density. The average electron density is thus obtained from the ideal gas law:

$$n_{e,av} = 2\frac{p_{vessel}}{k_B T_{vessel}}$$

where p_{vessel} and T_{vessel} are the pressure and the temperature in the vacuum vessel before the discharge.
Note that this does not take into account the impurity contribution. For a given gas pressure, a mixture of hydrogen with other gasses will likely produce more electrons than pure hydrogen, since an impurity atom will provide at least (and in general more than) one electron. The plasma electron density calculated as above is thus underestimated.
NB: Assuming a parabolic density profile of the form $n_e(r) = n_{e,0}\left(1-\frac{r^2}{a^2}\right)$, it is easy to calculate the relation between $n_{e,av}$ and $n_{e,0}$: $n_{e,av} = \frac{n_{e,0}}{4}$.

2.7. Safety factor

The safety factor (in general denoted *q*) is the number of toroidal turns of a field line necessary to complete one poloidal turn. Any two field lines on the same magnetic surface have the same safety factor, so *q* is defined for each magnetic surface. In the large aspect ratio approximation, the safety factor can be expressed as:

$$q(r) = \frac{rB_{Tor}}{RB_{pol}}$$

In this expression, *r* is the minor radius of the magnetic surface, $B_{Tor}(r)$ and $B_{Pol}(r)$ are the toroidal and poloidal magnetic field components averaged over the magnetic surface, and *R* is the major radius of the considered surface. The only unknown in this expression is $B_{Pol}(r)$. All the other quantities are measured.

→ Reminder on Ampère's law (https://en.wikipedia.org/wiki/Amp%C3%A8re%27s_circuital_law) : denoting *I(r)* the current flowing through an electrical conductor of radius r and B the magnetic field, recall the expression of Ampère's law. In this expression, an integral appears over a closed path. Explain the shape of this loop.

The general expression of Ampère's law is:

$$\mu_0 I(r) = \oint B.dl$$

The integral is over a loop enclosing the electrical conductor. The scalar product means that only the component of B along the loop will play a role.

→ In order to determine $B_{Pol}(r)$, apply Ampère's law to the case of a tokamak plasma. In that case, *r* is the minor radius of the considered magnetic surface and the integral is over a loop of the same minor radius.

Let us apply Ampère's law to a closed poloidal loop encircling the magnetic axis at a distance r:

$$\mu_0 I(r) = \oint B.dl = \oint B_{pol}.dl = 2\pi r B_{pol}(r) \quad ,$$

where I(r) is the plasma current enclosed by the loop.

→ Using the previous results and the definition of the safety factor, calculate the safety factor at the last closed flux surface.

We can now express q(r) replacing B_{pol} with its expression as a function of I(r):

$$q(r) = \frac{2\pi r^2 B_{Tor}}{\mu_0 I(r) R} .$$

The edge safety factor is thus: $q_a = \frac{2\pi a^2 B_{Tor}}{\mu_0 I_P R}$. *It is of particular importance in tokamak experiments since it plays an important role in the MHD stability. If the edge safety factor is close to 2, it is almost impossible to have a stable plasma. In most tokamaks, $q_{edge} \sim 3$ is favourable to plasma stability, hence the name of 'safety factor'.*

2.8. Plasma energy content

The plasma energy content can be determined using the temperature and density estimated above. You probably remember that in a gas, temperature is defined so that the average kinetic energy of a molecule is $\frac{3}{2} k_B T$ (https://en.wikipedia.org/wiki/Temperature).

In a plasma this must be refined. Instead of molecules, we have ions and electrons. In general they do not have the same temperature, so we define the electron temperature T_e and the ion temperature T_i. These temperatures are not uniform in the plasma but they are uniform on a magnetic surface, so we must consider their radial profiles $T_e(r)$ and $T_i(r)$.

→ What are the electron energy and ion densities on a magnetic surface of minor radius *r*?

On a magnetic surface of radius r, let us denote the electron density $n_e(r)$ and the ion density $n_i(r)$. The electron and ion energy densities are $n_e(r) k_B T_e(r)$ and $n_i(r) k_B T_i(r)$ respectively.

→ What is the elemental energy in a small volume *dV* around a magnetic surface of minor radius *r*?

The number of electrons in a small volume dV around a magnetic surface is $n_e(r) \times dV$. In the same way, the number of ions is $n_i(r) \times dV$. Thus, the kinetic energy contained in this volume is $n_e(r) k_B T_e(r) dV + n_i(r) k_B T_i(r) dV = k_B [n_e(r) T_e(r) + n_i(r) T_i(r)] dV$.

→ What is the total kinetic energy content of the plasma?

The total plasma energy is thus the sum of the elemental energy over all magnetic surfaces:

$$W_k = \int_{V_P} n\left(kT_e + kT_i\right)dV \approx 2\int_{V_P} n_e kT_e dV$$ *(with $n = n_e = n_i$ and assuming $T_i \approx T_e$)*

With the same polynomial forms as above for the density and temperature profiles, the plasma energy writes:

$$W_k(t) \approx \pi^2 a^2 R n_{e,0}(t) kT_{e,0}(t) \; .$$

with W_k and $kT_{e,0}$ in J and $n_{e,0}$ in m^{-3}.

There is also a way to determine the plasma energy content from the magnetic measurements using the pressure equilibrium equation and Ampère's law. This method is used to give the value shown on the GOLEM shot results page.