

A Langmuir probe system incorporating the Boyd-Twiddy method for EEDF measurement applied to an inductively coupled plasma source.

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Abstract

This paper reports on a modern implementation of a Langmuir probe method that was first proposed in the 1950's. The system directly measures the first and second derivative of the $I-V$ characteristic and determines the EEDF. This probe offers many advantages over existing systems that rely on numerical differentiating and data smoothing techniques. In addition to the EEDF analysis the system also incorporates several theories to extricate the plasma parameters from the ion collection and electron retardation branches of the $I-V$ characteristic.

Introduction

The determination of low temperature plasma parameters by probes is one of the most often used and well known procedures in the field of plasma diagnostics. The method originally developed by Langmuir and Mott-Smith in the 1920s [1] has been enhanced and developed over the decades in order to extend the application of the method to a variety of plasma conditions. Experimental, computational and theoretical enhancements to the original procedure of Langmuir and Mott-Smith have all been aimed at extricating the actual plasma parameters from the $I-V$ characteristic of the probe with a greater degree of confidence.

Using Langmuir probes is relatively straightforward if the plasma under consideration is essentially Maxwellian in nature and the electron energy can be characterised by a single scalar T_e . In the case where a Maxwellian distribution function can be assumed then one or more of several different probe theories[1,2,3,4] that have been developed over the years can be applied, depending on the particular conditions of the plasma. However if additional electron populations are present, as in arc driven plasmas or if parts of the electron energy spectrum have been depopulated due, for example, to inelastic collisions then the plasma electrons are non-Maxwellian in nature and analysis of the plasma using these techniques yields misleading results [5].

In [1] Langmuir and Mott-Smith establish the relationship between the second derivative of the probe characteristic and the EEDF for a spherical collector. The important extension to the theory in the 1930's by Druyvesteyn [6] shows that the

EEDF is proportional to $\sqrt{V} \frac{d^2 I}{dV^2}$, where $d^2 I/dV^2$ is the second derivative of the probe current-voltage characteristic for any convex collector. This allows the determination of the EEDF from the I - V characteristic.

Starting with the expression,

$$I_e(V_{bias}) = eA \int_{-\infty}^{\infty} dv_x \int_{-\infty}^{\infty} dv_y \int_{v_{min}}^{\infty} dv_z f_e(\mathbf{v})$$

for electron current to a probe at a voltage $V_{bias} < V_p$ (the plasma potential) for an arbitrary electron velocity function, $f_e(\mathbf{v})$ where v_{min} is the minimum velocity an electron needs to reach the biased probe and x and y are the directions perpendicular to the probe, the Druyvesteyn analysis culminates in the following expression for the EEDF,

$$N(\epsilon) = \frac{2}{Ae^2} \left(\frac{2m\epsilon}{e} \right)^{\frac{1}{2}} \frac{d^2 I}{dV^2} \quad (1)$$

where $N(\epsilon)$ is the number of electrons with energy in the interval $[\epsilon, \epsilon+d\epsilon]$ eV, V is the probe voltage, ϵ is the probe voltage with respect to the plasma potential V_p , ($\epsilon = V_p - V$), A is the probe area, $d^2 I/dV^2$ is the second derivative of the current-voltage characteristic and e and m are the electronic charge and mass.

The electron density, n_e , is obtained from the integral of $N(\epsilon)$.

$$n_e = \int_0^{\infty} N(\epsilon) d\epsilon \quad (2)$$

In the case of a non-Maxwellian electron energy distribution the notion of electron temperature is ambiguous but nevertheless an effective electron temperature, analogous to T_e for the Maxwellian case, can be defined from the integral of the distribution function as,

$$T_{eff} = \frac{2}{3} \langle \epsilon \rangle = \frac{2}{3n_e} \int_0^{\infty} \epsilon N(\epsilon) d\epsilon \quad (3)$$

Knowledge of the EEDF in non-equilibrium processing plasmas and ion sources is important for many reasons, for example explaining the hysteresis observed in the E to H transition for inductive plasmas[7]. Our particular interest in the EEDF and our motivation for developing the method arises from the need to model negative ion production in neutral beam ion sources for nuclear fusion applications (heating and

diagnostics). The models require an accurate EEDF in order to calculate the rate coefficients for the production and destruction of the negative hydrogen or deuterium ions[8,9].

The Boyd-Twiddy Method

The most straightforward approach to measuring the EEDF is to take the IV characteristic and numerically differentiate twice while measuring the probe voltage with respect to the plasma potential. The plasma potential can be found from where the 2nd derivative equals zero. However this approach is inherently noisy and requires subjective choices to be made with regard to smoothing the data between successive differentiation operations and the outcome is subject to the choice of smoothing method employed [10].

In this paper we present an alternative method first used in the 1950's by Boyd and Twiddy [11]. The method is to superimpose a square wave (300 Hz) modulated sine wave (2000 Hz) voltage (e_m) on the probe voltage V ; the superimposed voltage can be represented as

$$e_m = E \left[\frac{1}{2} + \frac{2}{\pi} (\cos pt - \frac{1}{3} \cos 3pt + \dots, etc) \right] \cos \omega t \quad (4)$$

where E is the peak of the modulated signal, and p and ω are the frequencies of the modulation and carrier signals respectively.

Introducing this nonlinear perturbation on the probe voltage induces a nonlinear response in the current drawn by the probe. By Taylor's Theorem the probe current can be expressed as,

$$I = f(V + e_m) = f(V) + e_m f'(V) + \frac{e_m^2}{2!} f''(V) \dots$$

The component of current measured at frequency p receives contributions only from even-order derivatives, provided that ω is not a multiple of p . Demodulating the current to remove the component at the carrier frequency, reveals a component at frequency p that is predominantly proportional to the second derivative as required. The component of the current measured at frequency p is given by,

$$i_p = \left[\frac{E^2}{2!} \frac{1}{\pi} f''(V) + \frac{E^4}{4!} \frac{3}{8} \left(\frac{1}{4} + \frac{1}{\pi^2} \right) f''''(V) + \dots etc \right] \cos pt \quad (5)$$

Terms involving the fourth and higher order derivatives can be neglected, hence the second derivative may be obtained from a direct measurement of i_p . In practical units the EEDF is given by.

$$N(\epsilon) = \frac{8\pi}{A} \left(\frac{m\epsilon}{e^3} \right)^{\frac{1}{2}} \frac{i_p(rms)}{E^2} (\text{eV})^{-1} \text{cm}^{-3} \quad (6)$$

The effect of the finite value of the ac amplitude on the measurement of the 2nd derivative as reported in [12] was minimised by varying the value of E from 0.05V just below the expected plasma potential where the 2nd derivative is high to 1 V in the ion collection region where the 2nd derivative is small. This variation was done in an exponential fashion in order to optimise simultaneously the signal to noise ratio and the energy resolution. The system was tested on a variety of passive circuits with non-linear current-voltage characteristics and the instrument was found to reproduce faithfully (to within 1%) the expected 2nd derivative. In these tests a square wave modulation of a sine wave was found to be more accurate than a sine wave modulated sine wave due to increased contributions from higher order terms in the latter.

Hardware and Software

The novelty of the probe system reported in this paper is that it takes an idea from the 1950's based on analogue electronics and adapts it to take full advantage of the progress made in PC based data acquisition and signal generation. A detailed description of the hardware and software setup and the calibration system can be found elsewhere[13].

A single PCI card (the most widely used card format for PC's), contained within a desktop PC and operating under LabVIEW™, provides all outputs, data acquisition and analysis functions. The probe waveform produced via a bipolar power amplifier consists of a dc bias (typically -20 to 20 V) with a modulated ac signal (sine wave modulated sine wave) superimposed of variable amplitude. The ac current from the instrumentation amplifier is analysed using a software implemented fast Fourier transform (FFT) analyser. The rms value of the FFT coefficients at the carrier and modulation frequency are recorded. The phase of the current at the modulation frequency is also recorded. This is used to determine the plasma potential since at the plasma potential the 2nd derivative passes through zero, this corresponds to an abrupt

phase reversal of the current at the modulation frequency. The dc component is also recorded to demonstrate the conventional Langmuir probe trace and for system calibration. The system that we have developed is also capable of acting as a conventional Langmuir probe. When operating in conventional mode the data are analysed by the same software program

The traditional Langmuir probe I - V characteristic trace is analysed using the following methods.

- i. The Orbital Motion Limited (OML) theory of ion collection [1,3].
- ii. The Allen-Boyd-Reynolds (ABR) radial motion theory of ion collection [2].
- iii. Bernstein-Rabinowitz-Laframboise (BRL) theory of ion collection [4].
- iv. Classical Langmuir-Mott Smith (LMS) theory for electron collection [1].
- v. A generalised fit valid for spherical or cylindrical probes from reference[14]

In all cases the pertinent data are extracted from the trace and is fitted to the theory using the Levenberg-Marquart non-linear fitting method. Classical Langmuir theory determines an electron temperature based on the assumption that the electrons have a Maxwellian distribution. OML, BRL and ABR theories can be used to infer electron temperature but these methods are very unreliable as only a small portion of the electrons in the high energy range are considered. The energy distribution in this range is very likely to differ from the rest of the distribution unless there is a strong influence of electron-electron interactions. Hence it is preferable to use Langmuir theory to infer T_e and then use this value to fit the OML, BRL and ABR theories. The BRL theory is fitted to the data following a procedure described in [15].

The Source

The probe performance was tested with a well characterised inductively coupled plasma (ICP) source, operating at 27 MHz and power of 160 W-360 W. The source shown in figure 1 has a cylindrical geometry with a “stovetop” type with a water cooled antenna. The source is equipped with a cylindrical Langmuir probe of diameter 100 micrometers and length 5 mm. The experimental results presented in this paper pertaining to the Boyd-Twiddy method were obtained using an uncompensated probe, however the IV characteristic was identical to that obtained by the compensated probe. The source was operated in principally in Helium with

admixtures of hydrogen and deuterium. In all cases the probe is positioned in the middle of the chamber.

Results

Evaluation of the EEDF

The results presented in this paper are selected to give an overview of the performance of the Langmuir probe system when operated in a well characterised ICP source. Firstly a typical Boyd-Twiddy measurement is discussed and compared to the traditional method for evaluating the EEDF. Secondly three sets of EEDFs are presented to illustrate the evolution of the EEDF with RF power, gas pressure and Deuterium partial pressure. Thirdly we present a comparison of the plasma parameters (density and temperature) as determined by the integrals of the EEDF (equations 2 and 3) with the parameters as determined by the analysis of the IV curve using the theories listed above.

In Figure 2 typical data obtained with the probe is presented. In this instance the current is read 20 times at each voltage setting and then averaged, no further smoothing or filtering is performed to obtain the numeric results presented. The dc and modulated ac current are processed and recorded simultaneously. In figure 2(a) the IV characteristic is presented along with the first derivatives obtained by both the Boyd-Twiddy method and numerical differentiation. In this case the first derivative is obtained from measuring the component of the current at the carrier frequency. It is clear from the figure that the first derivative by the Boyd-Twiddy method is smoother than that obtained using unsmoothed numerical differentiation. The second derivative curves are shown in figure 2(b). In this case the Boyd-Twiddy curve remains smooth and relatively noise free up to the plasma potential compared to the numerical result that exhibits a large degree of noise in this region. The noise in the numerical calculation can be smoothed but smoothing the data in the exponential region inevitably leads to loss of detail in the measured distribution function. The plasma potential can be determined from where phase of the ac current to abruptly changes by 180 degrees. This gives good resolution to the measured plasma potential since the amplitude of the modulation is as low 0.05 V at this point.

The second derivatives as determined by both methods are converted to EEDFs using the Druyvesteyn formula and presented in figure 2(c). In this figure the EEDF as evaluate by three methods are presented. The curve labelled BT is the result of the

Boyd-Twiddy method, the curve labelled “Hybrid” is the result of numerical differentiation of the 1st derivative (i.e. the data obtained from the analysis of the component of the current at the carrier frequency ω). The curve labelled “numerical” is the EEDF obtained from the numerical 2nd derivative of the dc component of the IV characteristic. Each representation of the EEDF is obtained by averaging 20 IV characteristics. In the interest of clarity error bars are shown on alternate points. The error bars on figure 2c are a convolution of the standard deviation of the 20 measurements at each point and the uncertainty resulting from the differentiation method. The figure illustrates the benefit of the Boyd-Twiddy method in this case in terms of the reproducibility of the data. Over 20 successive measurements the standard deviation of the Boyd-Twiddy method is less than 8% at all points over 2 orders of magnitude. This compares with 12% for the hybrid and 65% for the numerical method.

The results presented are all derived from measurements made with a modulation frequency of 300 Hz, at this frequency the system requires approximately 10 ms per data point. The system has been tested and performed well with modulation frequencies of up to 6 kHz. (the limit being imposed by the bandwidth of the instrumentation amplifier.)

The evolution of the EEDF at fixed power for various pressures is presented in figure 3. This data as well as the data in figures 4 and 5 tend to be good quality up to approximately 20 eV. However noise from the instrumentation amplifier dominates the modulation frequency at this level.

In figure 4 the evolution of the EEDF with input power is shown. The pressure in this case is 20 Pa and the gas is a 10:90 deuterium to helium mixture. The form of the EEDF remains essentially maxwellian over the range of power from 160 W to 350 W. Figure 5 shows how the form of the EEDF changes as the Deuterium content is increased from 10% of the mixture to 90 % of the mixture. In this figure the distribution function has been normalised to the electron density as the powers are slightly different for each data set. However the form of the EEDF is clearly seen to change from a single temperature maxwellian to a form more closely resembling a bi-maxwellian distribution with a high energy tail. This can be explained by the difference in electron kinetics in He and D₂ (or H₂) plasmas[16]. Vibrational excitation of the D₂ molecules is an effective energy loss mechanism for electrons between 3 eV and 10 eV. Therefore as the D₂ concentration is increased this

mechanism becomes more dominant and the resulting departure from the Maxwellian form becomes more pronounced as is evident from figure 5. Similar behaviour was observed with admixtures of H₂ to He.

Plasma Parameters

The electron retarding branch is fitted to the Langmuir & Mott-Smith theory of electron collection using the non-linear least squares Levenberg-Marquart technique. In the program the theory is fitted using the two free parameters of electron density and electron temperature and the third parameter of plasma potential is obtained from the derivatives of the curve as measured using the Boyd-Twiddy technique. The results of this procedure presented in figure 6(a) is found to fit the data very well provided a correction is made for the ion current in the region negative of the floating potential. This correction is made by subtracting the ion current, extrapolated by means of a fit to one of the ion collection models, from the probe current. In the case of the data presented here the OML model was chosen for the correction, however there was little difference between this and a correction made using the ABR or BRL models.

The electron density and electron temperature (T_{eff}) as determined by the integrals of the EEDF are not subject to any fitting routine but are sensitive to the choice of plasma potential. While the values obtained from the integrals do not differ substantially from those obtained from the IV analysis it should be noted that this is predominantly Maxwellian plasma hence they are expected to be similar. In the case of the deuterium concentration scan it is found that as the integral values progressively differ more as the departure from the Maxwellian EEDF increases.

The negative side or ion branch of the IV curve is fitted to the various ion collection theories using the same Levenberg-Marquart technique below -5 volts. Generally it is found that the BRL theory provides the best fit for the data, this is not surprising since the BRL theory is the most complete and also the most difficult to implement in the software! The residuals for the various fits are shown in figure 6(b). Regarding the ion branch, the curve is fitted to the theories using the density and plasma potential as free parameters, the electron temperature is determined from the electron branch analysis. The goodness of fit is quite insensitive to the chosen value of the electron temperature but this fitting is not particularly good if the plasma potential as determined from the electron branch is used as a fixed parameter. In all cases the fits to all the ion collection theories applied are much improved if they are fitted with

using a plasma potential that is about 5 volts lower than the electron branch measured plasma potential. This suggests that the probe itself is perturbing the plasma, causing an increase in the plasma potential as the probe bias is increased and more current is drawn from the plasma. The EEDF is likely to be underestimated and inaccurate from 0-5 eV because of this effect.

In figure 7 the electron temperature is shown as a function of pressure in the range 10 Pa to 20 Pa. The narrow range of operating pressures was due to the limitations of the matching network. The values of T_e and T_{eff} agree to within 15% and the trend is for the temperature to decrease with increasing pressure. This trend can be explained in self sustaining ICP's by the fact that the increased collisionality reduces the ion diffusive loss. This leads to a lessening of the ionization rate required to sustain the current driven by the power coupled into the plasma.

The plasma density as a function of pressure is plotted in figure 8. The different values of the density shown in the figures are determined by fitting the IV characteristic with the two theories for the ion collection branch (negative biased end) and two theories for the electron retarding branch (positive biased end) of the curve. Generally, all of the ion collection theories agreed with each other to within 5%. However there is a large discrepancy between both values derived from the electron branch and the values derived from the ion branch. The discrepancy between the BT values and the Ion collection theory values is most likely attributable to the uncertainty in the potential of the probe relative to the plasma potential. The shifting plasma potential can be a particularly problem in sources that have built up an insulating layer on the walls of the source. A solution currently being implemented in the next version of this probe is to incorporate into the system a reference probe to track any changes in the plasma potential. This is an option that has been successfully implemented in many probe systems for ICP measurements [17]. The discrepancy between the values obtained using L-MS theory and the BT method can have a number of origins. Determining electron density from the LM-S theory is prone to errors due to sheath expansion effects caused by probe geometry or thermal effects caused by departure from thermal equilibrium. Generally it is better to use the ion branch for density determination and use the electron branch to determine the electron temperature determination [18,19]. Sheath expansion will cause the electron saturation current to increase and hence cause the electron density to be overestimated using L-MS theory alone. However if the sheath area increases linearly with probe

voltage then the BT method has the advantage in that, by nature of the direct measurement of the second derivative involved, it is immune to additional linear terms in the IV characteristic as well as being immune to departures from thermal equilibrium.

Discussion

A PC based Langmuir probe system has been developed based on the Boyd-Twiddy technique but adapted to take advantage of modern computer processing power. The probe measures the 1st and 2nd derivative of the IV characteristic directly and determines the EEDF from the Druyvesteyn formula. The analysis software includes several theories for the analysis of the ion and electron branch of the IV characteristic. The probe has previously been used to determine the EEDF in three different sources, a high density arc driven source [9], a high density RF ICP source operating at 1 MHz and a pulsed ECR source [20]. In this paper we report on the use of the probe in a low power RF ICP operating at 27 MHz. In all cases the EEDF measured agrees well with that determined from numerical differentiation but the data are less noisy. However in the case of the ECR source best results often tended to be the EEDF obtained from the “Hybrid” procedure (i.e. numerical differentiation of the Boyd-Twiddy derived 1st derivative). This was due in part to the fact that the source was pulsed in the kHz range.

The main advantage of the Boyd-Twiddy technique is that it gives reasonably unambiguous representation of the EEDF from inexpensive multi-purpose data acquisition cards and measurement equipment. The LabView™ software that drives the system is standard in most laboratories.

The main disadvantage of using the Boyd-Twiddy method is that time taken to acquire the full EEDF data is long in comparison to other commercially available systems. The time taken to acquire an EEDF depends on the modulation frequency chosen. Using a modulation frequency of 300 Hz a reasonably good EEDF with energy resolution of 0.5 eV and range of 25 eV can be acquired approximately 0.5 seconds or about three modulation periods per point. Faster measurements can be made by increasing the modulation and carrier frequency. A less accurate measurement can be made using the hybrid approach, in this case the time taken is 3 carrier frequency periods per point. The system is unsuitable at present for pulsed

plasma measurements, however a boxcar acquisition approach would be relatively easy to implement.

The benefit of the technique is clear in the results presented in this paper. Whether or not this Boyd-Twiddy implementation is better in general than alternative systems, where efforts are made to improve electronic sampling and digital filtering prior to numerical differentiation, is debatable. To determine the benefit accruing from the system vis-à-vis the alternatives and under what conditions any benefit may be gained, requires a more comprehensive evaluation such as that described in reference [10].

The present limitations of the probe as an EEDF diagnostic for sources in the density range 10^{10} - 10^{11} cm^{-3} is the noise from the amplifier that limits the dynamic range. This will be improved with a higher spec amplifier with a higher bandwidth and better signal to noise ratio.

ACKNOWLEDGEMENTS

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Figure Captions

Figure 1.

Schematic diagram of the ICP source used to evaluate the performance of the probe.

Figure 2.

(a) IV characteristic with first derivatives. (b) 2nd derivatives of the IV characteristic as determined by the Boyd-Twiddy technique and numerical differentiation. (c) The EEDF as evaluate by three methods: Boyd-Twiddy, Hybrid and numerical differentiation.

Figure 3.

The EEDF plotted for different gas pressures for a 10% D₂–90% He plasma. The rf power for the scan was 250 W.

Figure 4.

The EEDF plotted for different input powers for a 10% D₂–90% He plasma. The total pressure was 20 Pa.

Figure 5.

The EEDF plotted for different D₂ concentrations. The plasma conditions for this data were: Pressure=20 Pa.

Figure 6

(a) The IV characteristic fitted with curves for the Langmuir & Mott-Smith theory of electron collection. (◇) Data, (—) Langmuir & Mott-Smith (*electron branch*)
(b) The residual of the data fitted to different theories of ion collection. (Δ) OML theory, (□) ABR theory, (◇) BRL theory (ion branch)

Figure 7.

Electron Temperature measurements as a function of pressure. (◇) T_{eff} as evaluated from the integral of the EEDF and (□) T_e as determined from Langmuir theory. The source conditions for this data were: Power=250 W and gas: 10% D₂–90% He

Figure 8.

Plasma density measurements as a function of pressure. (◇) n_e as evaluated from the integral of the EEDF and (□) n_e as determined from Langmuir theory, (Δ). n_{ion} as determined from OML theory and (×) n_{ion} as determined from BRL theory. The source conditions for this data were: Power=250 W and gas: 10% D₂–90% He.

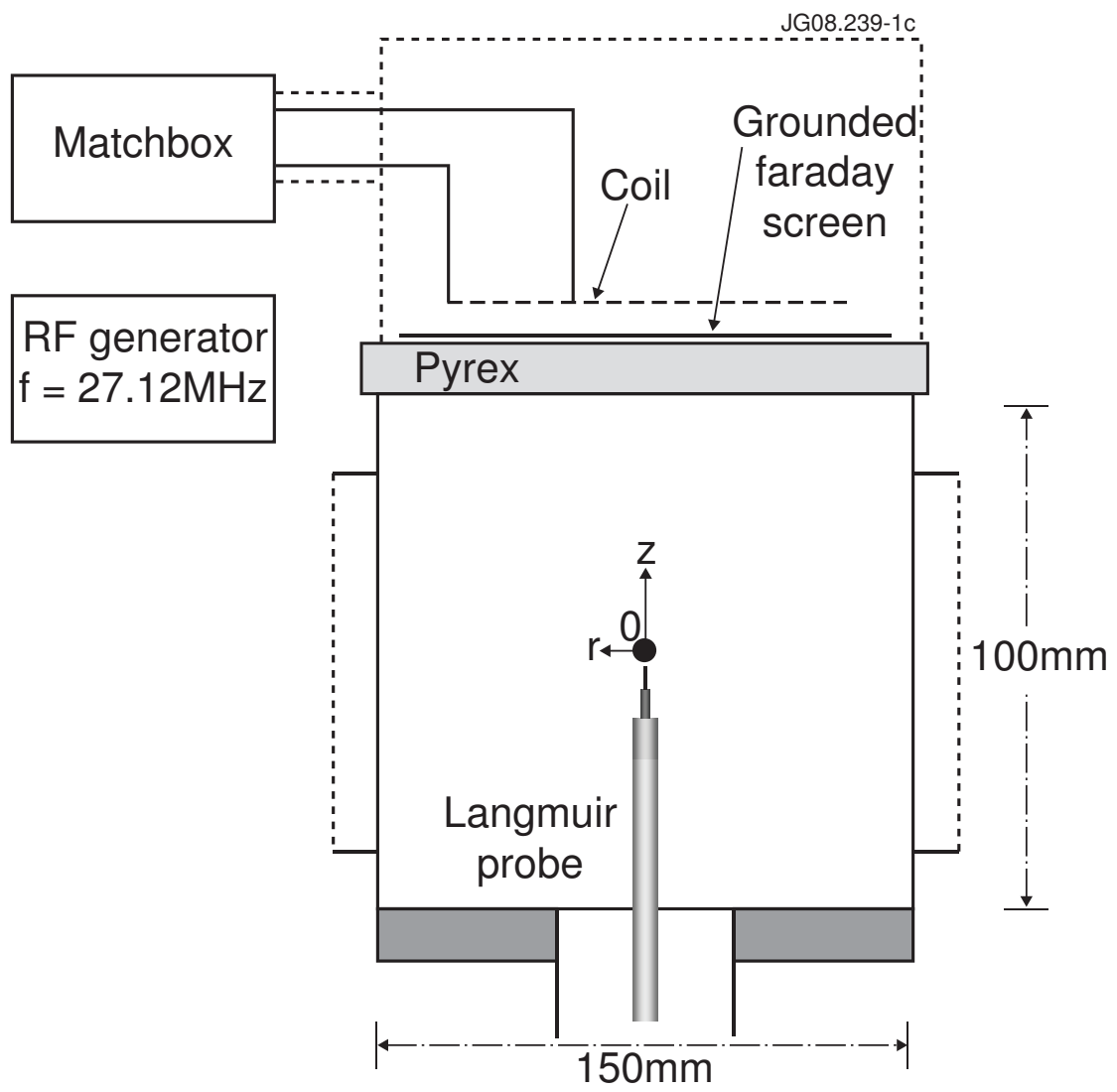


Figure 1

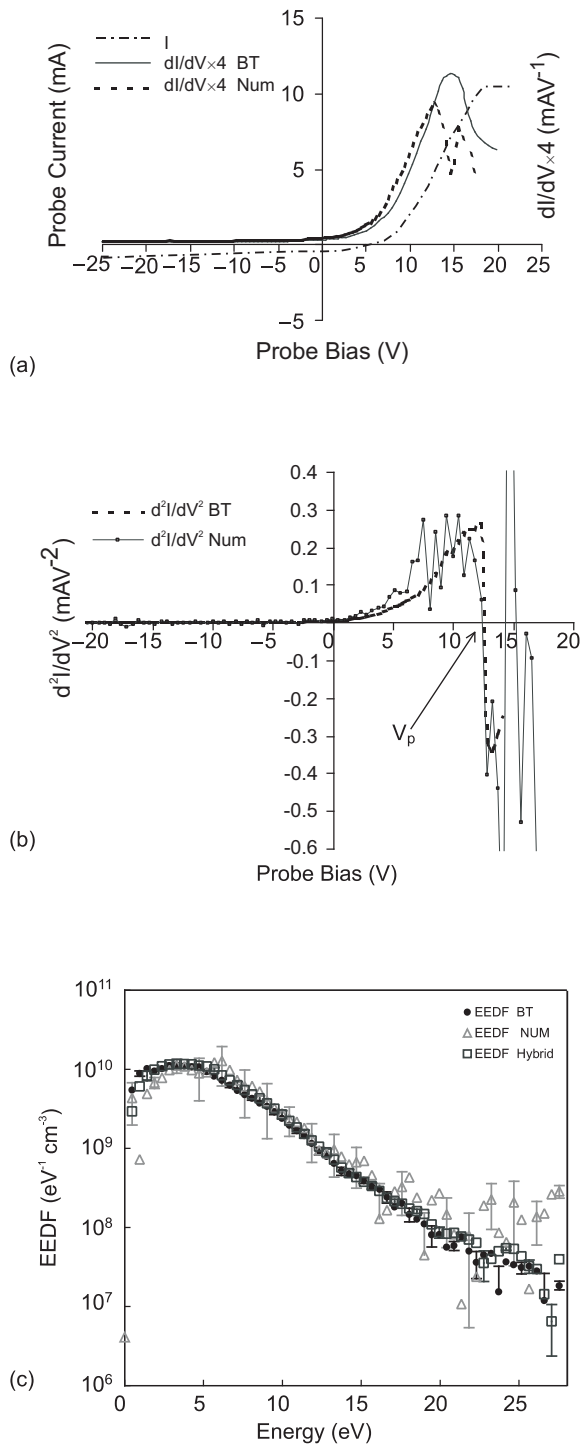


Figure 2

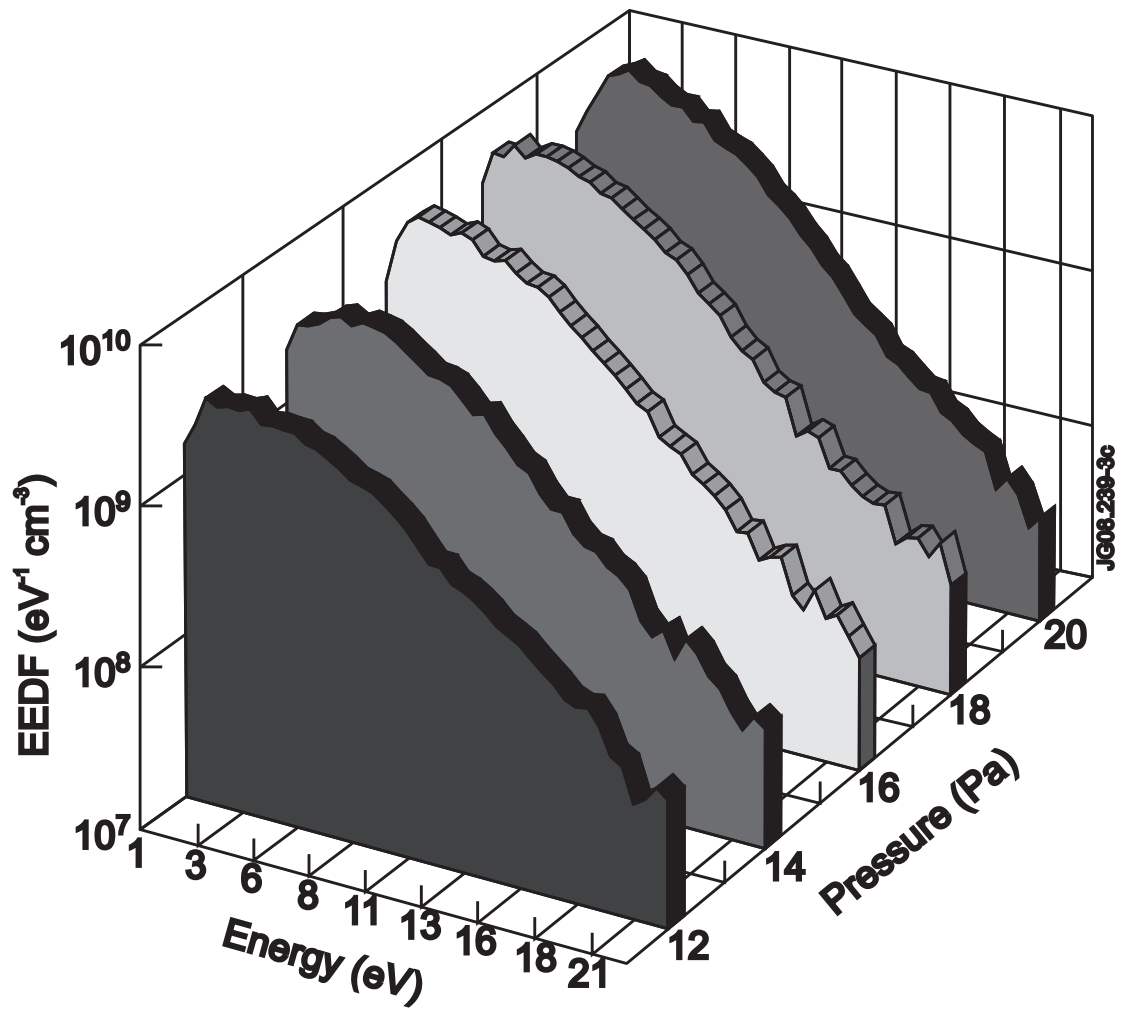


Figure 3

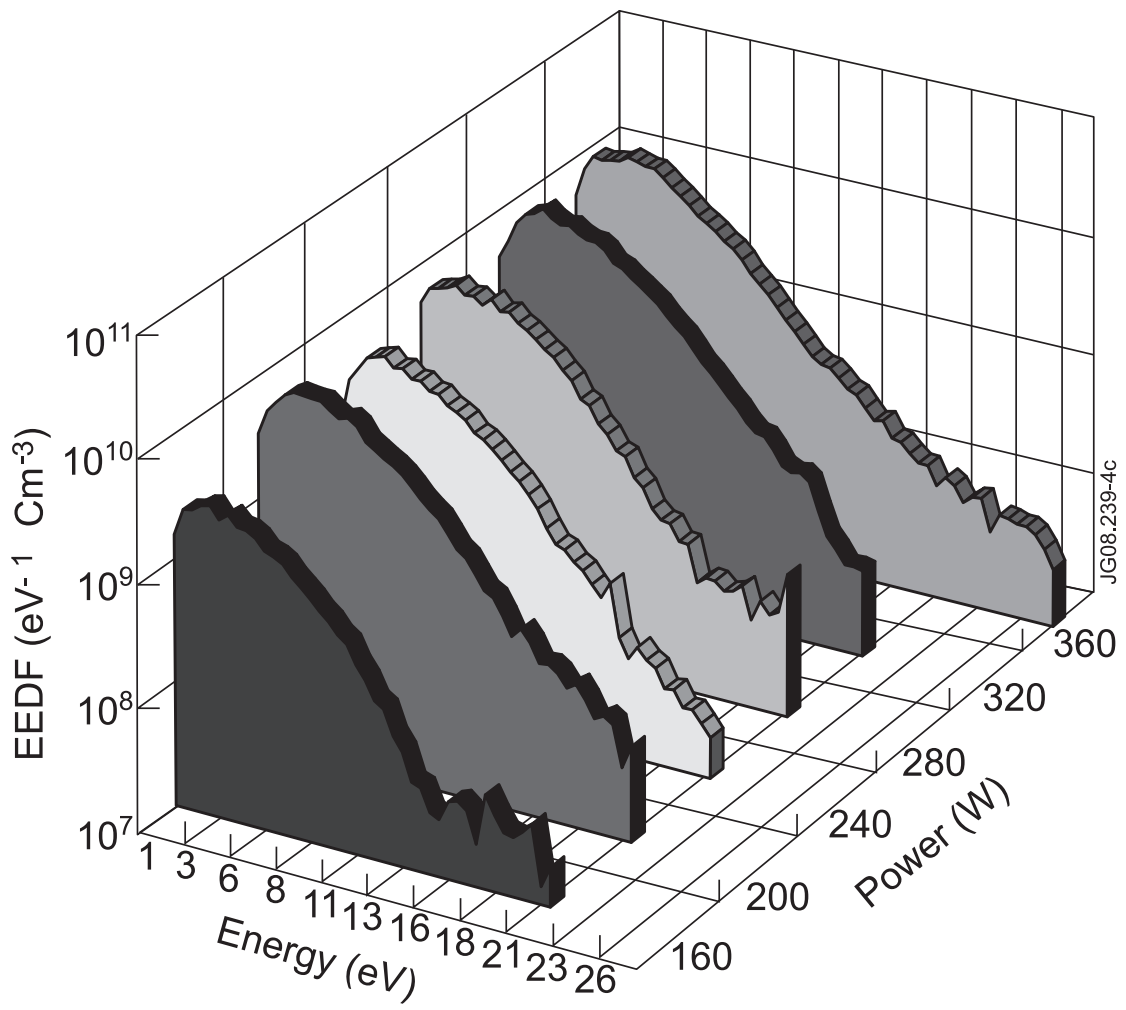


Figure 4

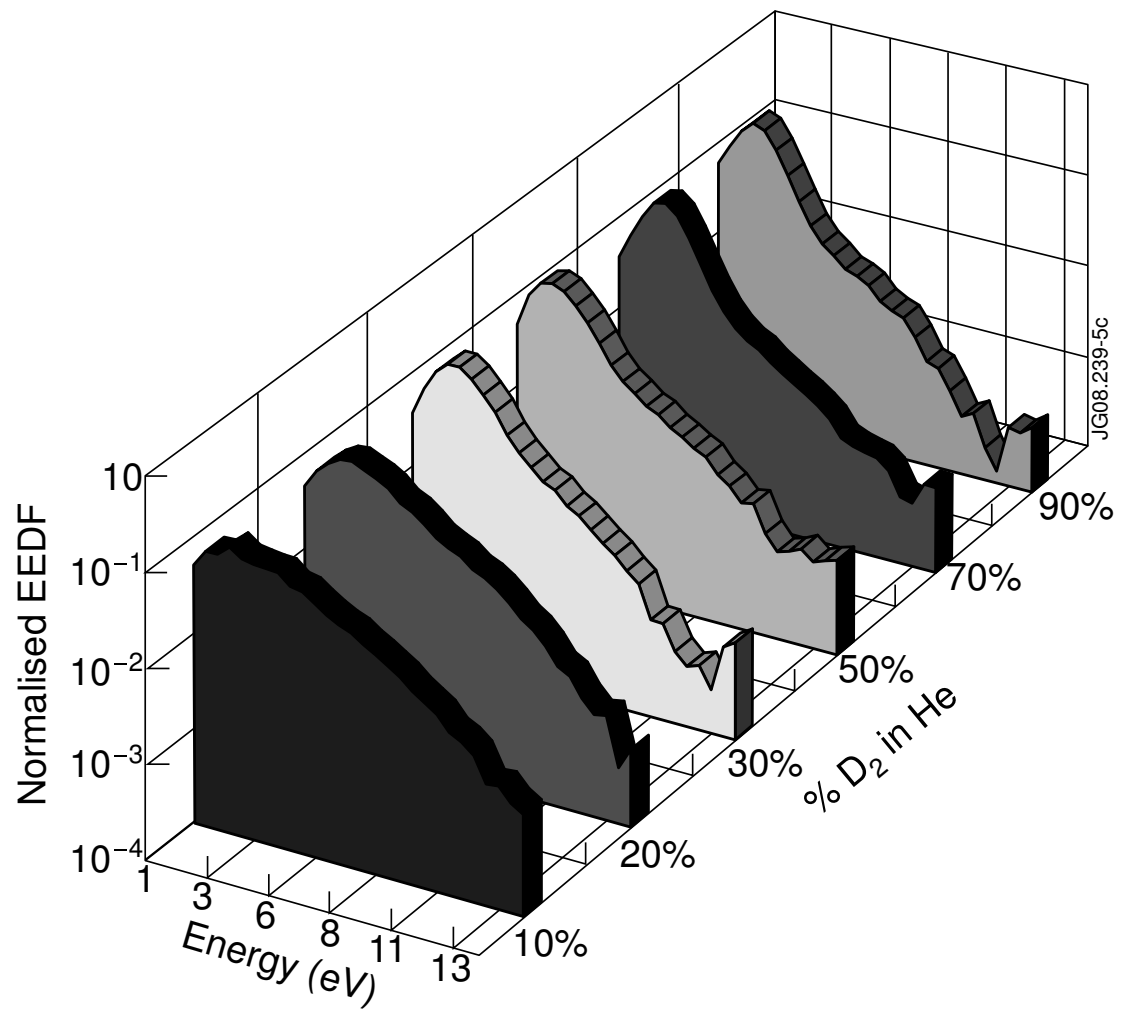


Figure 5

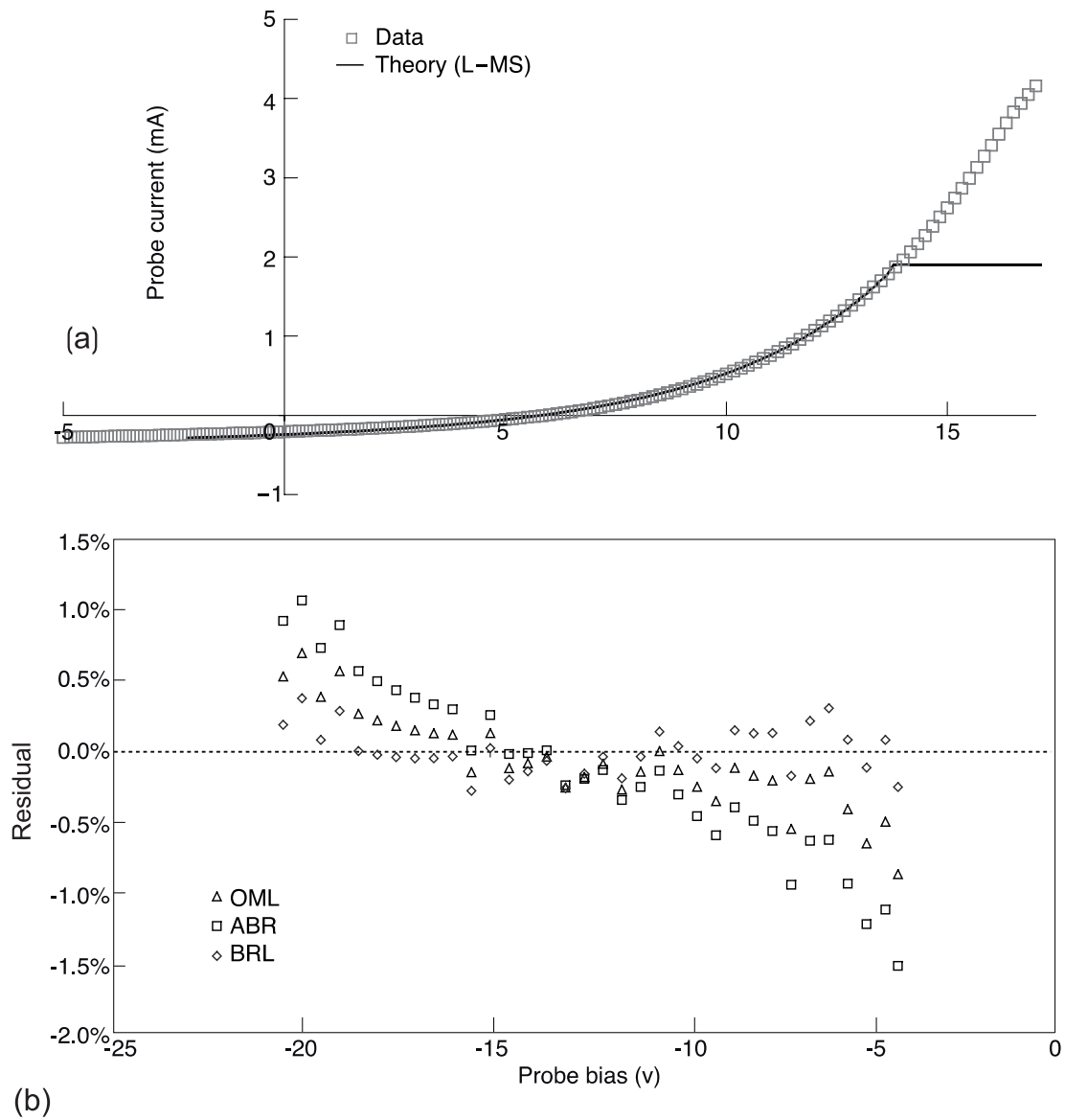


Figure 6

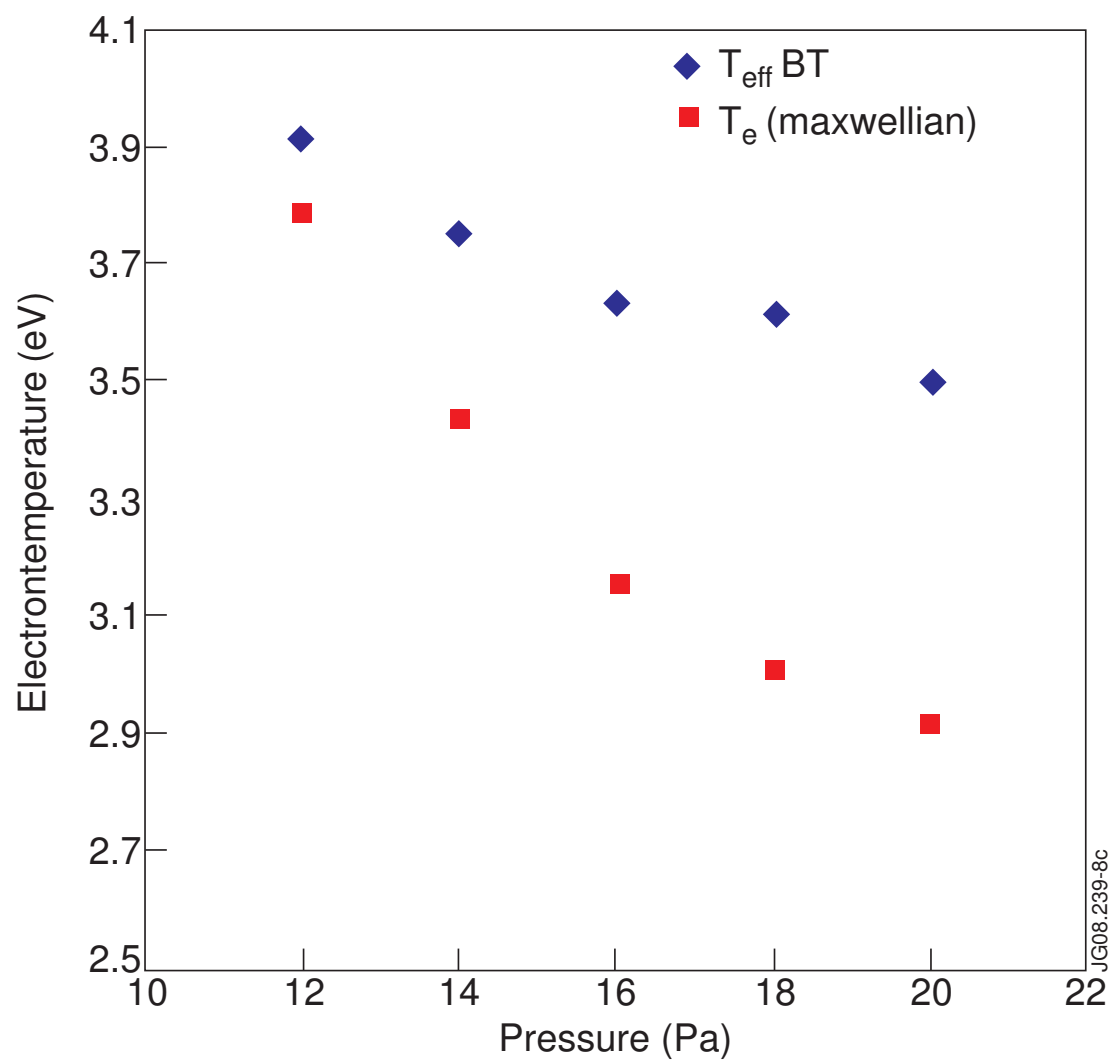


Figure 7

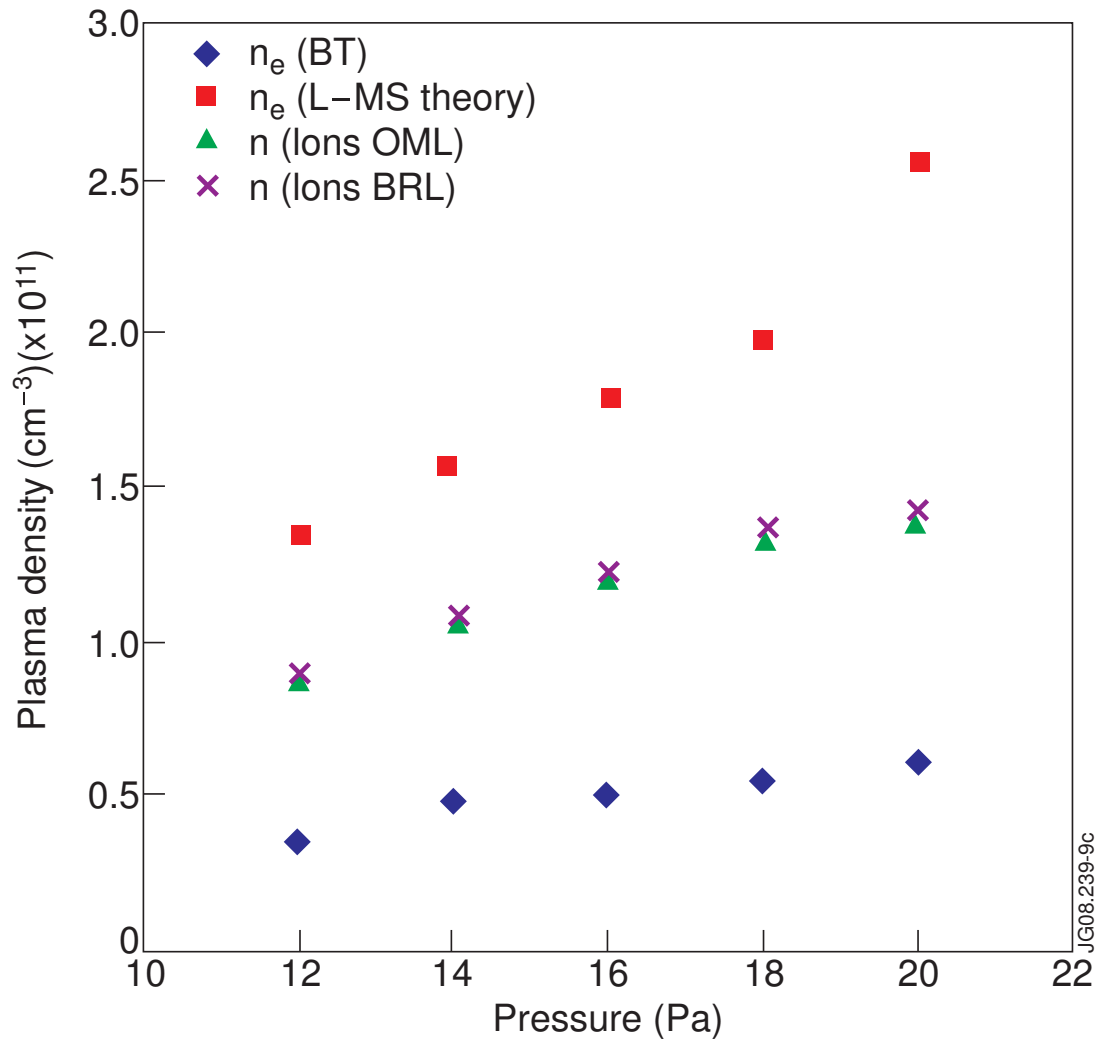


Figure 8

JG08.239-9c

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