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Development of 3D ferromagnetic model of tokamak core with strong toroidal asymmetry

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HIGHLIGHTS

- Full 3D core model was developed, as a continuation of the previous paper on ferromagnet modelling by the collective of the authors.
- Model characterizes the effect of whole volume of the 3D geometry ferromagnet as bound currents on its surface.
- Linearized form of the model is benchmarked on tokamak GOLEM, which features strong asymmetry of its iron core.
- 3D tokamak core model is compared to toroidally axisymmetric equivalent of tokamak GOLEM. 3D model successfully characterized main features of distortion of magnetic field by asymmetric tokamak core structure.
- For some specific locations, only qualitative correspondence between modelled and measured magnetic field was obtained. This discrepancy might be attributed to effects of partial saturation of the ferromagnet. Future work will be focused on implementation of non-linearity effects into the model.

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ABSTRACT

Fully 3D model of strongly asymmetric tokamak core, based on boundary integral method approach (i.e. characterization of ferromagnet by its surface) is presented. The model is benchmarked on measurements on tokamak GOLEM, as well as compared to 2D axisymmetric core equivalent for this tokamak, presented in previous work. Linearized model well describes quantitative characteristics of B_R field, generated by poloidal field coils located close to core central column, and distorted by ferromagnet. A discrepancy is seen between linearized form of model for B_R field generated by coils under the transformer limbs and the measurements. Future work will thus include implementation of the non-linearity effects in order to further investigate this issue.

1. Introduction

In many older tokamaks, including the currently largest operating tokamak JET, iron core is still used to support the transformer action. While this lowers energetic demands on the current drive system, it also introduces non-linearity and limits operation by eventual saturation of the core. Moreover core affects magnetic field in its vicinity and thus also plasma equilibrium and stability. While, axisymmetric iron core approximation is sufficient for reconstruction of toroidally symmetric equilibrium [1–4], to quantify effect of iron core on toroidally asymmetric plasma equilibria or fields (such as those generated by *EFCCs* for ELM mitigation experiments on JET [5]) a full 3D core model is necessary.

While specific 2D axisymmetric core geometries can be treated analytically [6–8], general 3D geometries need to be simulated. However, if point of calculation is located far from ferromagnet, the standard approach by differential finite-element method might have limited applicability, since in such a case large number of calculation elements is necessary as well as boundary conditions might be hard to define [9]. This can be avoided by using boundary integral equation method, where only ferromagnetic elements are defined and which is valid on infinite domain [9,10]. This method was successfully applied before to model behavior of 2D

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axisymmetric tokamak core equivalents [2,3,11] as well as of far 3D magnetic field of power transformer in [9].

This paper presents first results of application of this method to model 3D magnetic fields in close proximity of tokamak iron core numerically, as a next step of development of open-source 3D model of tokamak JET core. Following section describes the basic physical principles of the used method, while description of benchmarking device and experimental arrangement is provided in Section 3. Correspondence of model to experimental data is discussed in Section 4 with summarization in Section 5.

2. Ferromagnetic model by magnetization currents

The main principle of used model is to project all the volumetric effects of the ferromagnet onto its surface, such that its boundary is sufficient for its characterization. This is possible for homogeneously magnetized medium, represented as a set of elementary magnetic dipoles with macroscopic *magnetization vector* $\mathbf{M}$ [A m^{-1}]. Quantity corresponds to surface *bound current* density σ:

$$\sigma(\mathbf{r}) = \mathbf{M}(\mathbf{r}) \times \mathbf{n}(\mathbf{r}), \tag{1}$$

although it should be kept in mind that it does not represent an actual charge transfer. Normal vector $\mathbf{n}$ points outwards of the ferromagnet. By imposing condition that no currents (other than those of magnetization) are driven on the ferromagnetic surface, Ampére's law yields continuity of tangential component of magnetic intensity $\mathbf{H}_i$ on its both sides [4]:

$$\mathbf{n}(\mathbf{r}) \times (\mathrm{H}_0(\mathbf{r}) - \mathrm{H}_1(\mathbf{r})) = 0, \tag{2}$$

where index 0 represents air, index 1 ferromagnet and vector $\mathbf{r}$ position of their boundary. Substitution of Eq. (2) into Eq. (1) yields [12]:

$$\frac{\mu_0}{2}\boldsymbol{\sigma}(\mathbf{r}) = \lambda(\mathbf{r})\mathbf{B}(\mathbf{r}) \times \mathbf{n}(\mathbf{r}), \tag{3}$$

with $\lambda(\mathbf{r}) = (\mu_r(\mathbf{r}) - 1)/(\mu_r(\mathbf{r}) + 1)$ and $\mu_r(\mathbf{r})$ being relative permeability of ferromagnet. Quantity $\mathbf{B}(\mathbf{r}) = \mathbf{B}_{ext}(\mathbf{r}) + \mathbf{B}_{core}(\mathbf{r})$ represents total magnetic field, by both the external current sources (such as power coils and plasma) and the bound currents $\boldsymbol{\sigma}(\mathbf{r}')$ (with $\mathbf{r}' \neq \mathbf{r}$). Therefore [10]:

$$\sigma(\mathbf{r}) - \frac{\lambda(\mathbf{r})}{2\pi}\int_S \left(\sigma(\mathbf{r}') \times \frac{\mathbf{r}-\mathbf{r}'}{|\mathbf{r}-\mathbf{r}'|^3}\right) \mathrm{d}\mathbf{S}' \times \mathbf{n}(\mathbf{r}) = \frac{2\lambda(\mathbf{r})}{\mu_0}\mathrm{B}_{\mathrm{ext}}(\mathbf{r}) \times \mathbf{n}(\mathbf{r}). \tag{4}$$

By representing the core boundary as a set of N different elementary surfaces, a set of $2N$ equations is obtained:

$$\boldsymbol{\sigma}_k^i + \lambda^i \sum_{j \neq i}^{N} \sum_{l}^{2} (A_l^{ij} \boldsymbol{\sigma}_l^j) = \lambda^i C_k^i. \tag{5}$$

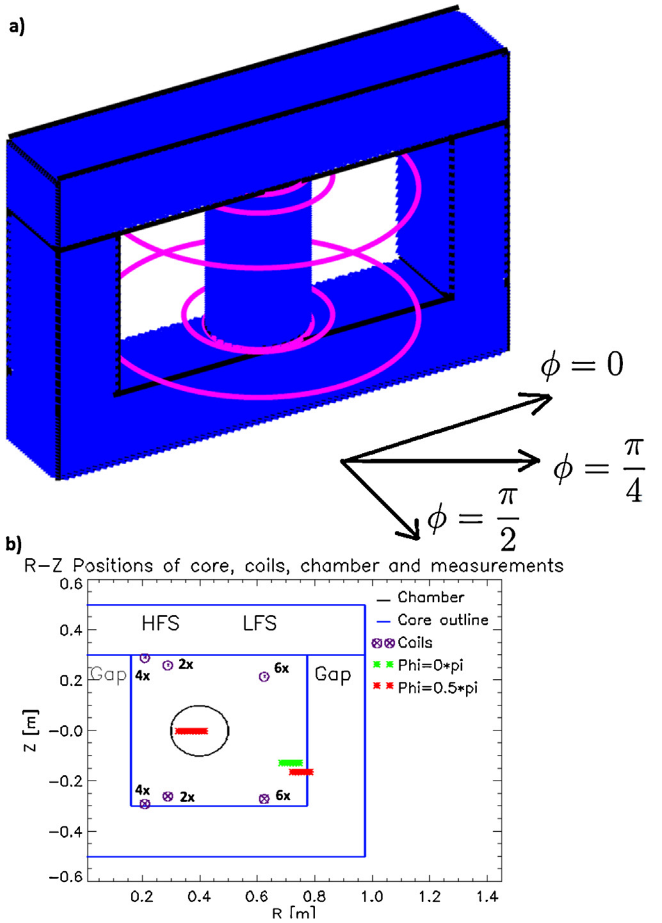

Fig. 1. (a) General shape of ferromagnetic core of tokamak GOLEM with directions of chosen values of toroidal angle ϕ and windings used for B_R field generation (purple circles). (b) *R–Z* plane cut at $\phi = 0$ toroidal position, including outline of chamber, polarities, turn numbers and *R–Z* positions of coils and of B_R scan measurements. $\phi = 0.5$ positions inside chamber correspond to both plots of Fig. 2. *HFS* represents the *high-field side*, while *LFS* the *low-field side*.

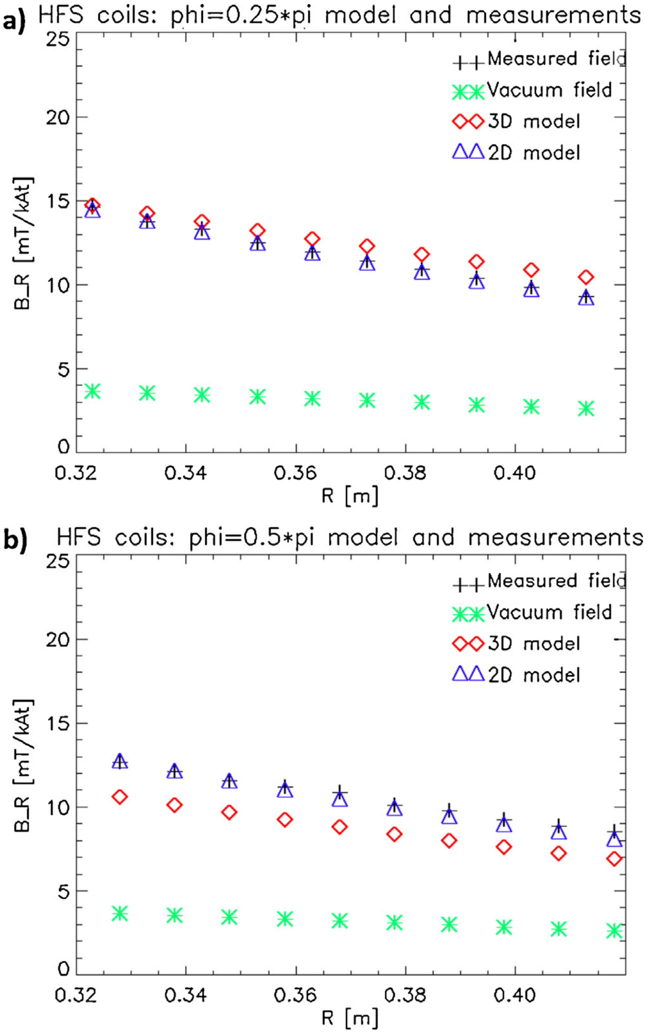

Fig. 2. B_R vs R scan measurements and models inside tokamak chamber on (a) $\phi = \pi/4$ and (b) $\phi = \pi/2$ positions. B_R generated only by set of 4 coils on *HFS*.

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Lower index refers to possible directional components with respect to plane orientation of elementary surfaces and upper index refers to the location of the element on the core. It should be also noted, that Eq. (5) in itself is non-linear, due to $\mu_r(\mathbf{r}) = f(\mathbf{B}(\mathbf{r}))$ dependency. Assumption of linearity is thus equivalent to assumption of non-saturation of ferromagnetic elements.

3. Applied iron core geometry and experimental arrangement

For benchmarking, the iron core of the tokamak GOLEM was used. The tokamak GOLEM [13] is the former CASTOR tokamak, with main parameters of $R = 0.4$ m, $a = 0.085$ m, $B_\phi < 0.5$ T, $I_p < 8$ kA. Measurements are represented by series of vacuum discharges, where only coils of vertical plasma stabilization (i.e. those of B_R field generation) were energized. Components of generated magnetic field were measured by Hall probe on long horizontal manipulator, capable of measurements of 3D fields [14]. Core and coil geometry, as well as R–Z locations of the measurements are shown in Fig. 1.

Measurements were carried out in two different configurations. Firstly, only the 4 *HFS* coils close to core central column were energized and magnetic field was measured inside the chamber only. Port availability limited ϕ of the measurements to $\phi = \pi/2$ and $\phi = \pi/4$ positions. This arrangement was also used to benchmark 2D form of core model in [11], with results also shown in Fig. 2 for reference. Secondly, the 2 *LFS* coils were energized and measurements were carried out in close proximity to core limbs (and on corresponding R–Z positions far from core), as to maximize 3D effects of the core.

Both experimental arrangements were chosen to prevent core saturation, so that linearized form of Eq. (5) was used in the model (i.e. $\lambda^i = 1$). Justification of this choice is discussed in Section 4. Since its elements were rectangular, model was able to retain (crude) shape of the real core (including the transformer gaps). This was not the case for the toroidally axisymmetric model of [11], where core was represented as large central column cylinder with two thick discs above and below, and with its elements being thin cylinder coats. Both the results of [11] (shown in Fig. 2) as well as new measurements (shown in Fig. 3) were evaluated using the 3D model. Additionally, for better overview of field manitude distribution, R–Z profiles of $|\mathrm{B}_\theta| = \sqrt{B_R^2 + B_Z^2}$ of *LFS* configuration for $\phi = 0$ and $\phi = \pi/2$ are shown in Fig. 4(a) and (b) respectively.

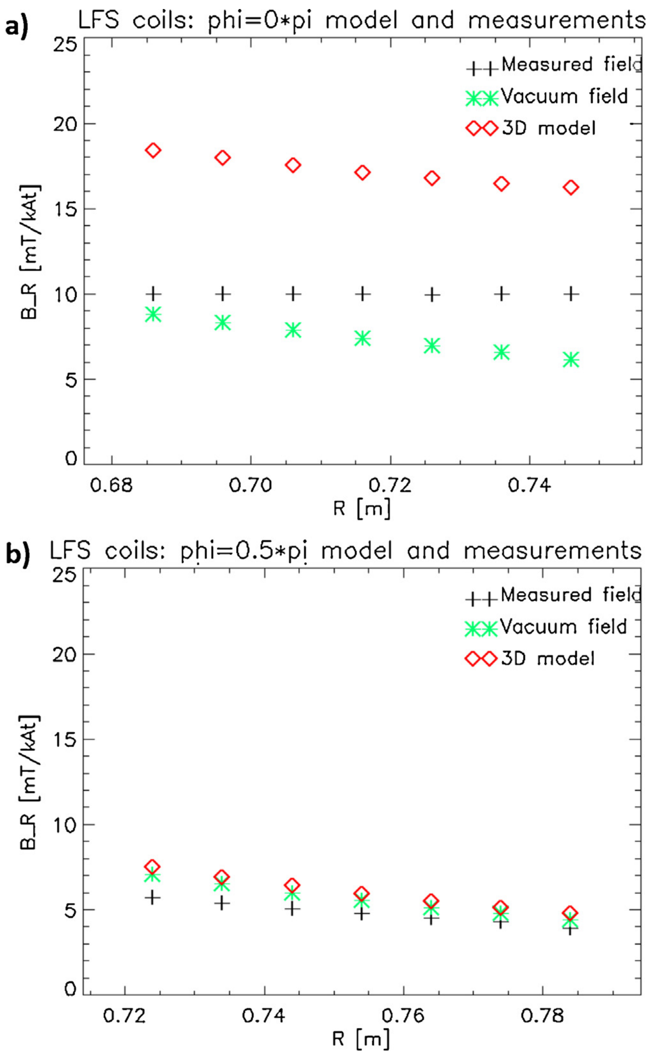

Fig. 3. B_R vs R scan measurements and models inside tokamak chamber on (a) $\phi = 0$ and (b) $\phi = \pi/2$ positions. B_R generated only by set of 4 coils on *LFS*.

4. Discussion of model applicability

As can be seen in Figs. 2 and 3 the B_R measurements (when compared to vacuum field approximation) show non-negligible field distortion by ferromagnetic core in both *LFS* and *HFS* configurations. Although effect on *HFS* coil configuration might not be that surprising due to very close proximity of power coils to central column of the core [11], the observed non-negligible toroidal modulation for *LFS* configuration underlines the necessity to use 3D core model.

Fig. 2 also shows that 3D model has successfully reproduced toroidal modulation of *HFS* coil B_R field. The slight discrepancy with respect to measured values might be caused by rough shape of core model, which was used due to lack of core technical documentation and bad accessibility to central column area (for measurements of

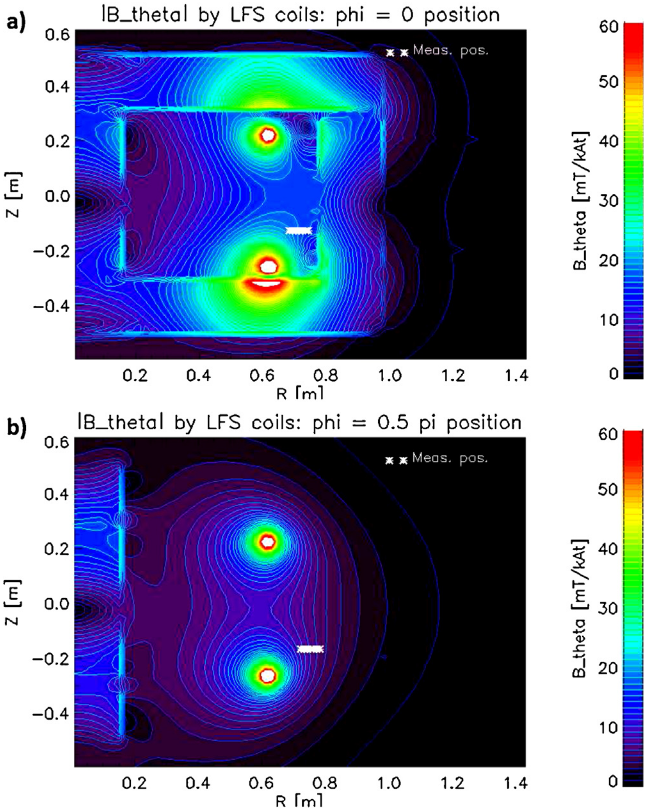

Fig. 4. R–Z profiles of modelled $B_\theta = \sqrt{B_R^2 + B_Z^2} = |\mathbf{B}|$ on (a) $\phi = 0$ and (b) $\phi = \pi/2$ toroidal positions. Field is generated solely by *LFS* position coils.

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core dimensions). For 2D model results, it should be noted that for each ϕ position the axisymmetric core equivalent of different dimensions was used (see [11]), in contrast to single, globally valid, predictive 3D model.

Fig. 3(a) indicates that linearized 3D model offers limited applicability for *LFS* configuration in $\phi = 0$ position, with predicted field being in average 70% higher in magnitude with respect to measurements. Enabling partial saturation of core components will however lower the magnitude of modelled field and thus implementation of non-linearity will be the main focus of future work. Nevertheless, even linearized model seems to successfully characterize some specific features of core field distortion. Specifically, measurements in Fig. 3(a) shows a flat $B_R(R)$ character. Modelled R–Z profile in Fig. 4(a) shows existence of such an area slightly above the location of the measurements. Non-linearity implementation is expected to shift this area downwards, which might then coincide with the measurement positions. Lastly, note that Fig. 3 does not feature comparison of 2D model to measurements, since such model is not applicable for *LFS* configuration.

5. Summary

A boundary integral method model of strongly asymmetric tokamak core was presented and benchmarked on tokamak GOLEM. The method itself projects all the volumetric effects on the ferromagnet surface and is not numerically demanding in its linearized form (which corresponds to unsaturated ferromagnet). The benchmarking included both most recent measurements of core-distorted tokamak field, as well as those presented in [11]. Model describes well the qualitative characteristics of magnetic field distortion by ferromagnet. For quantitative description the implementation of non-linearity might be necessary, which will thus be the main focus of future work, together with creation of satisfactory model of tokamak JET core.

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