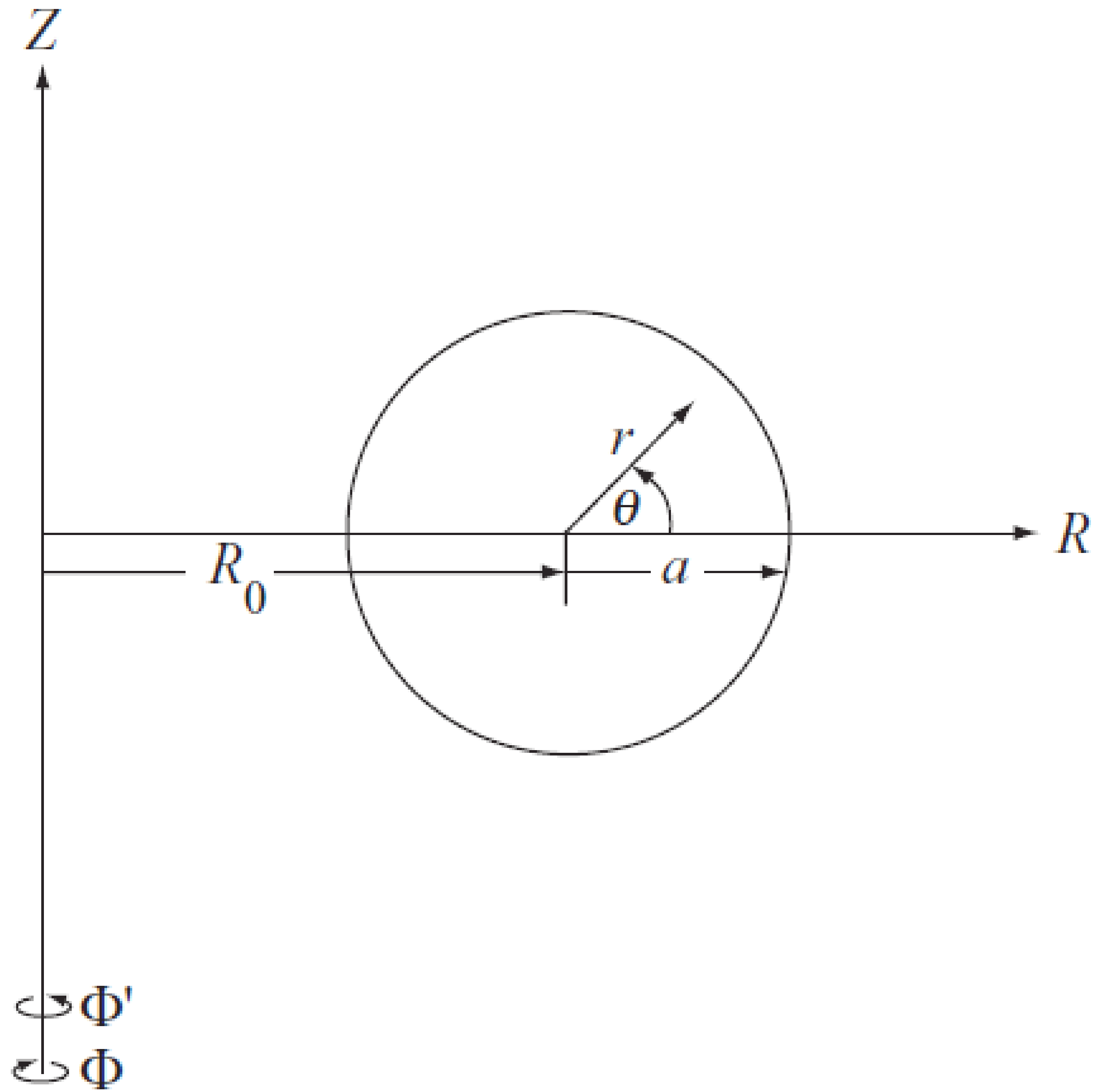
Plasma MHD Activity Observations via Magnetic Diagnostics

Magnetic islands, statistical methods, magnetic diagnostics, tokamak operation

Outline

- Summarization of island model and theory
- Application of statistical methods
  - FFT and spectrogram
  - Cross-correlation
- Summary of statistical methods

Used coordinate system

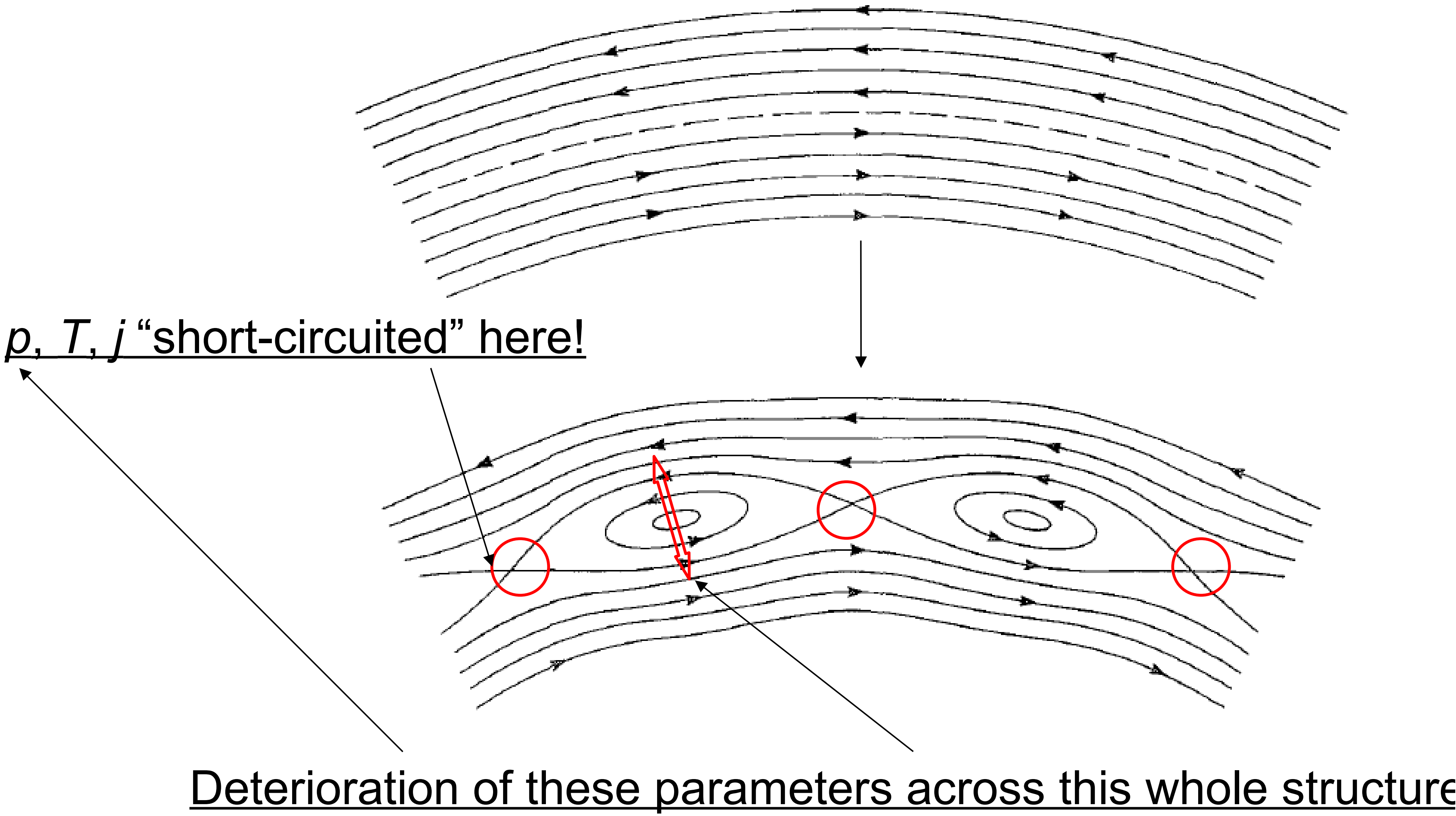


Safety factor

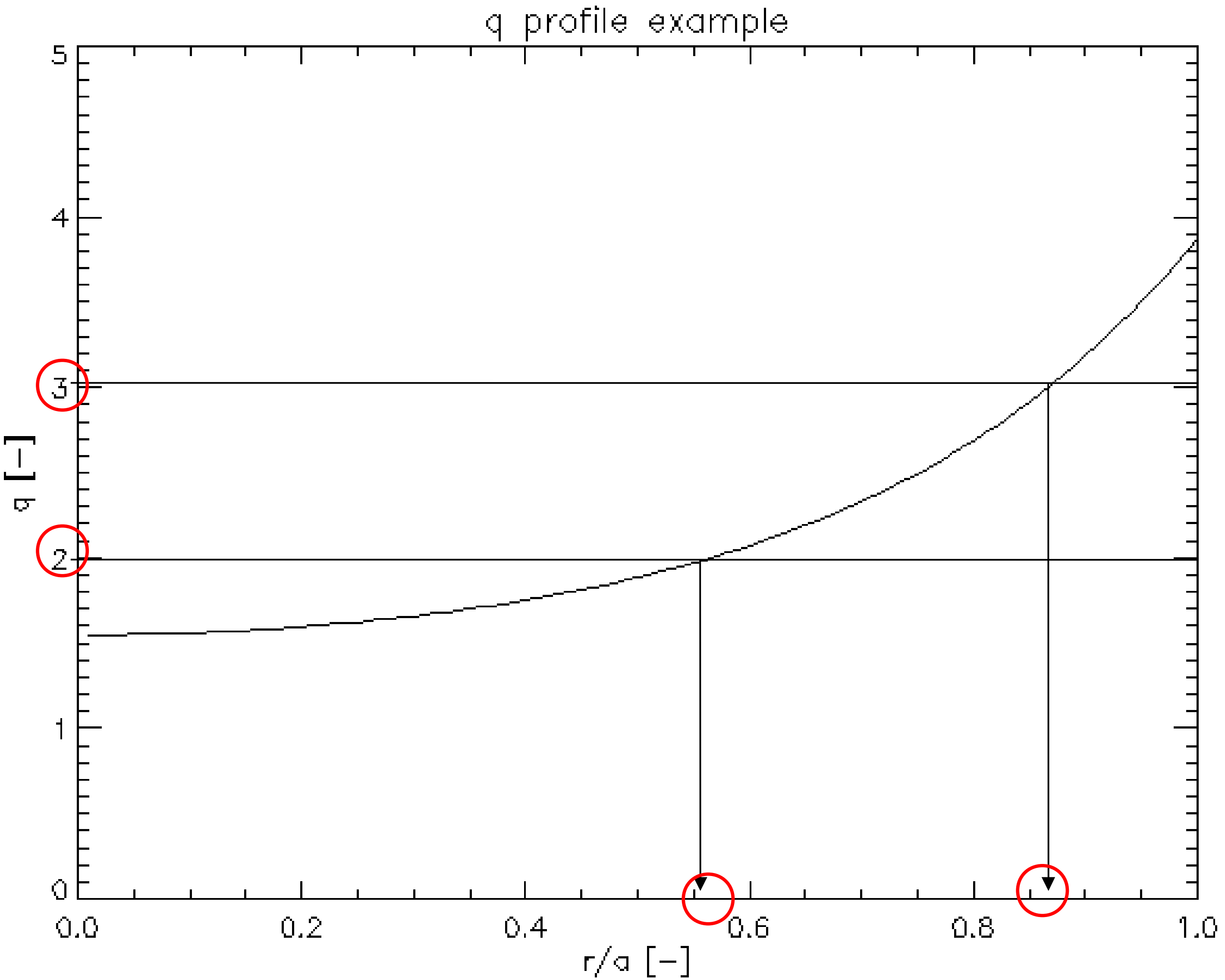
- Radial profile:

$$q(r,\nu) = \frac{2\pi B_T}{R\mu_0 I_p} \frac{r^2}{1-\left(1-\frac{r^2}{a^2}\right)^{\nu+1}}$$

- On *r* where *q* = *m*/*n* a perturbation of nested poloidal flux surfaces will emerge
  - On flux surface – *p*,*j*,*T* are constant


p, T, j “short-circuited” here!
Deterioration of these parameters across this whole structure

Locations where islands will emerge



Magnetic island model

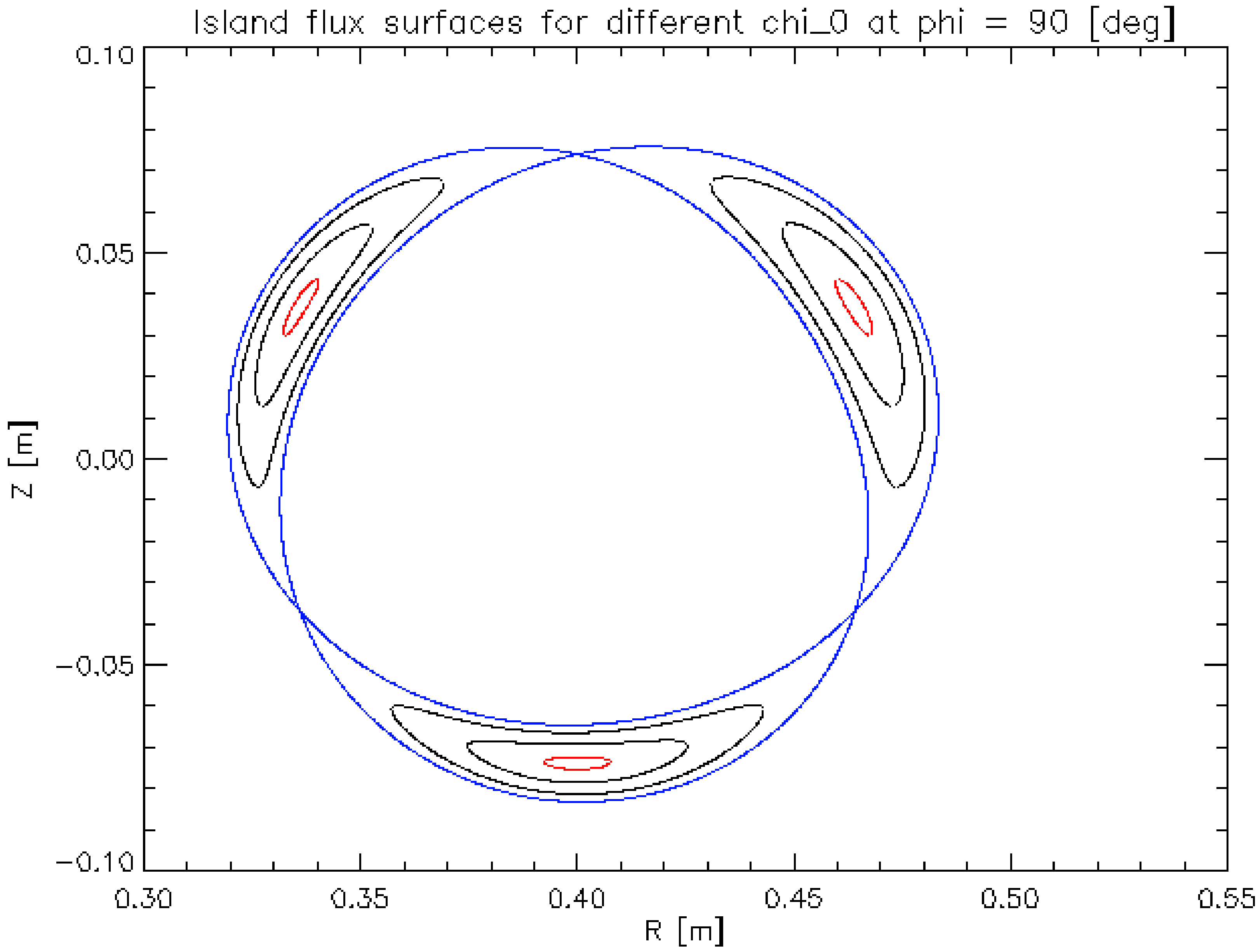
- From perturbed field line trajectory:

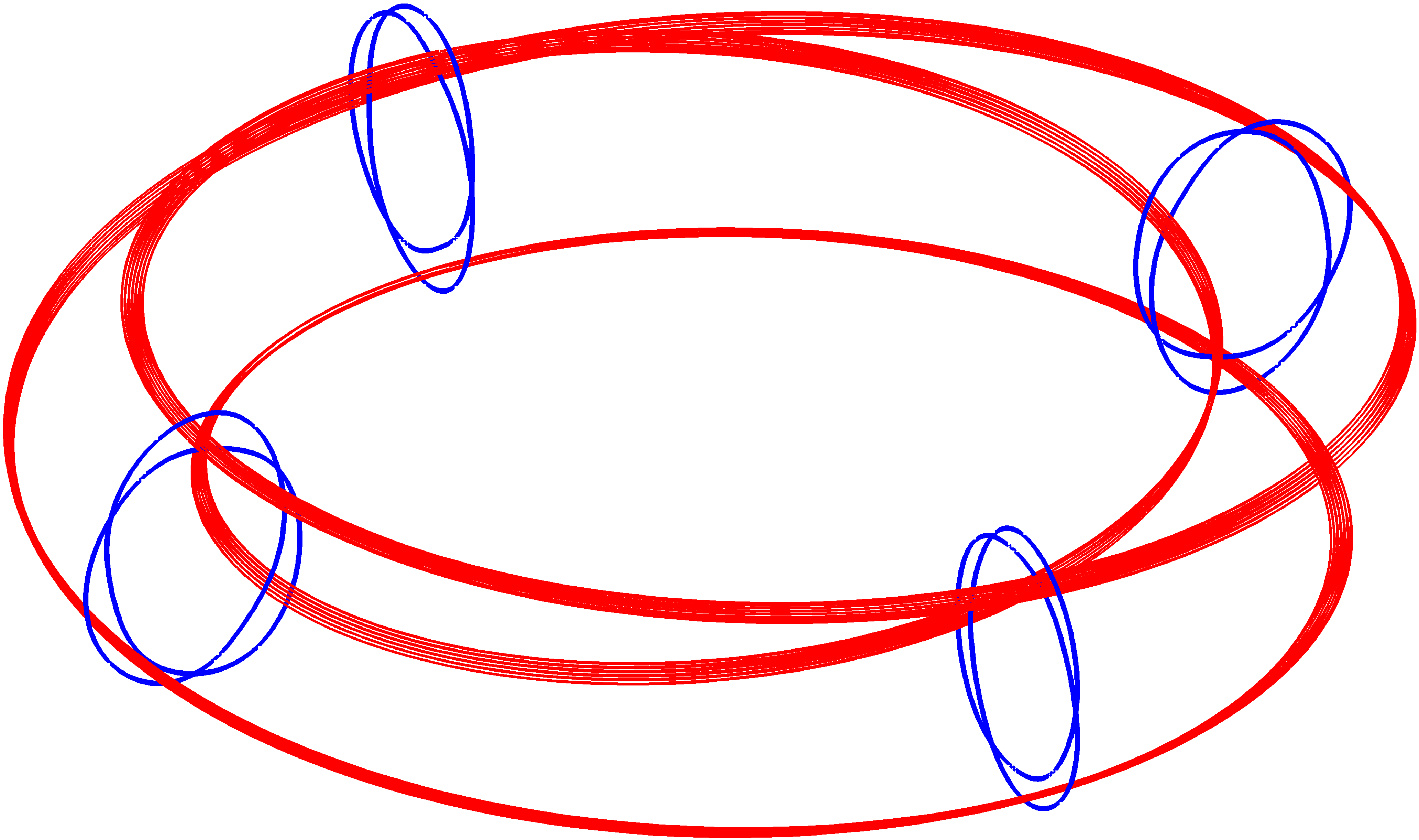
$$z^2 = \frac{w^2}{8}\left(\cos(m\chi) - \cos(m\chi_0)\right)$$

- With $\chi = \theta - \frac{n}{m}\phi \qquad z = r - r_s$

$$w = 4\left(\frac{r_s \widehat{B}_r q}{m q' B_\theta}\right)^{1/2}_{r_s}$$

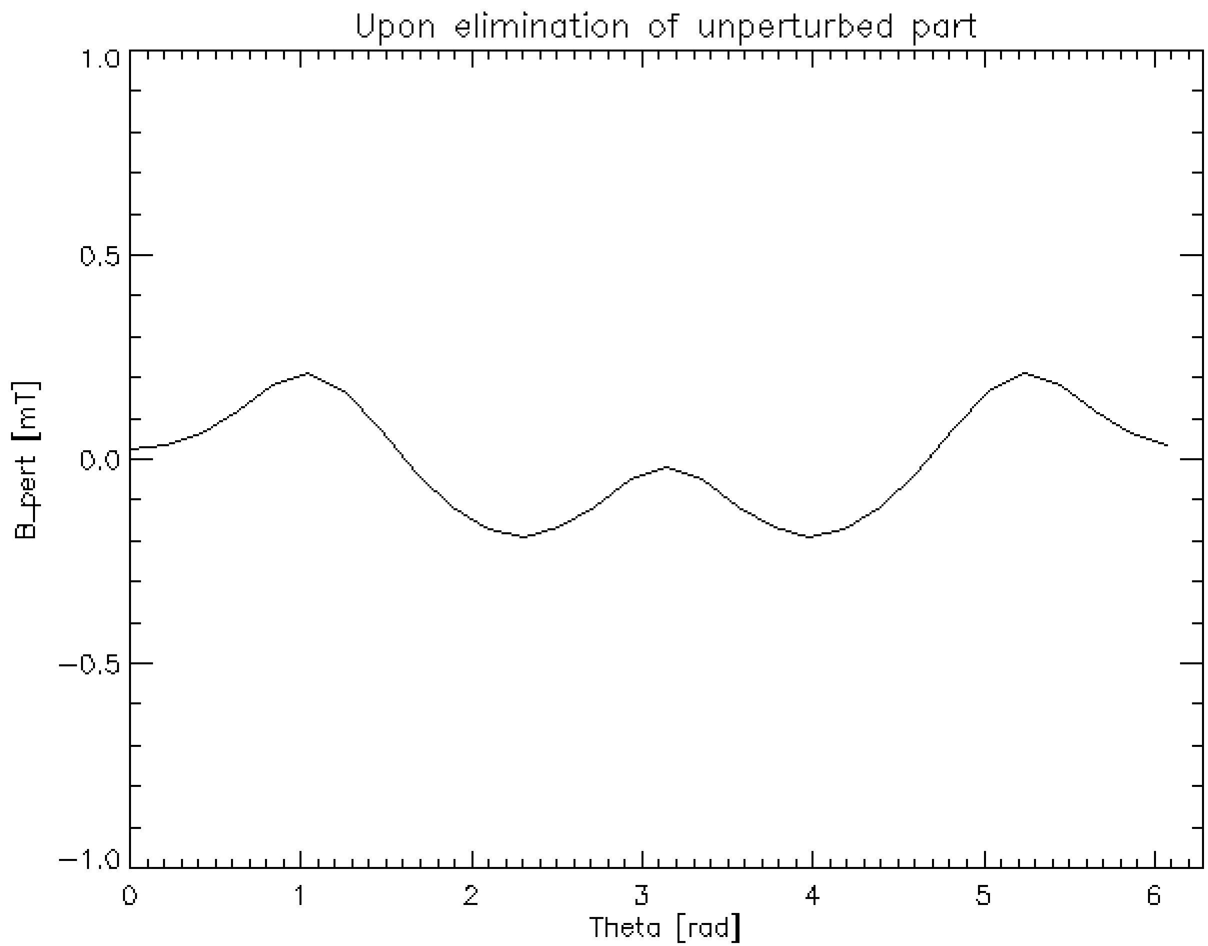
Typical GOLEM island



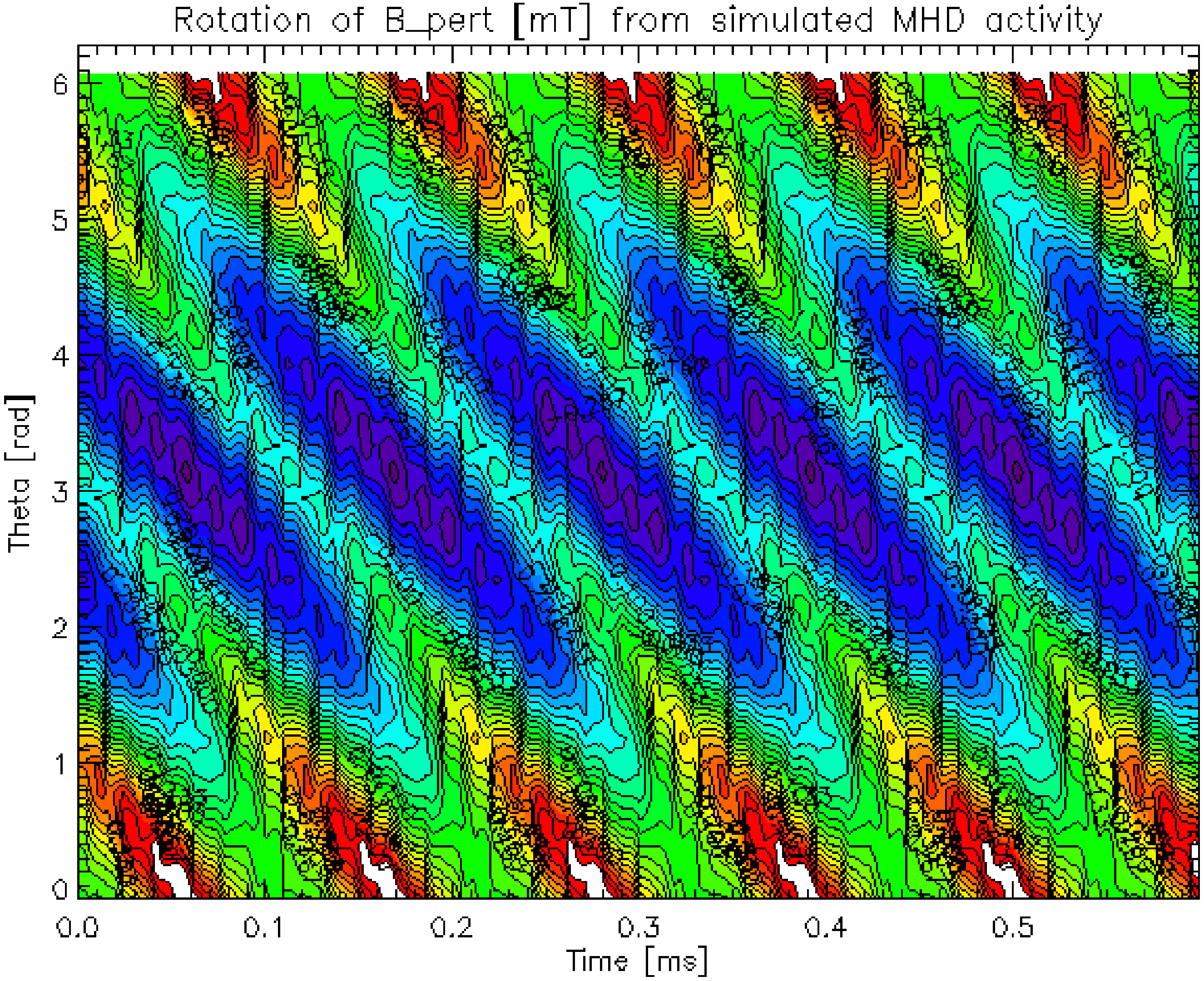


- Short-circuited *j*(*r*) profile by island
  - Poloidal magnetic field generated by plasma is slightly perturbed across poloidal angle θ
  - This depends on *m*-mode number of island

Signature of $m = 3$ island: perturbations of B_pol across θ



- Plasma and island rotates – *B_pol*(θ) changes with time
- Example of m = 3 island signal across (θ,t) space at 3 kHz poloidal rotation (by model):



Application of statistical methods

- Up until now – model
  - Known *m* and *f* of rotation
- Our task – opposite character
  - We are trying to identify *m* and *f* from data
- Analysis of temporal and spatial domain of signal – identification of *f* and *m* respectively
- To understand how is output of statistical methods of analysis connected to these quantities – application to known data from model

Fast Fourier Transform (FFT)

- FFT – discrete Fourier transform by character
- However, full Fourier transform:

$$B(\nu) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} B(t) \mathrm{e}^{-\mathrm{i}2\pi\nu t} \mathrm{d}t$$

Just how is one supposed to represent infinity
by finite interval of measurement?

- Solution – analyzed part of signal is assumed to periodically repeat from infinity to infinity

- This assumed periodicity introduces another issue:
  - If data do not start and end at 0 value – there will be infinite number of discontinuities
  - Representation of discontinuity by Fourier transform is possible
  - However, infinite number of coefficients is necessary to do so
  - That is not possible with finite number of data points
- Therefore – any discontinuity at start and end of analyzed time window must be eliminated first

Windowing method

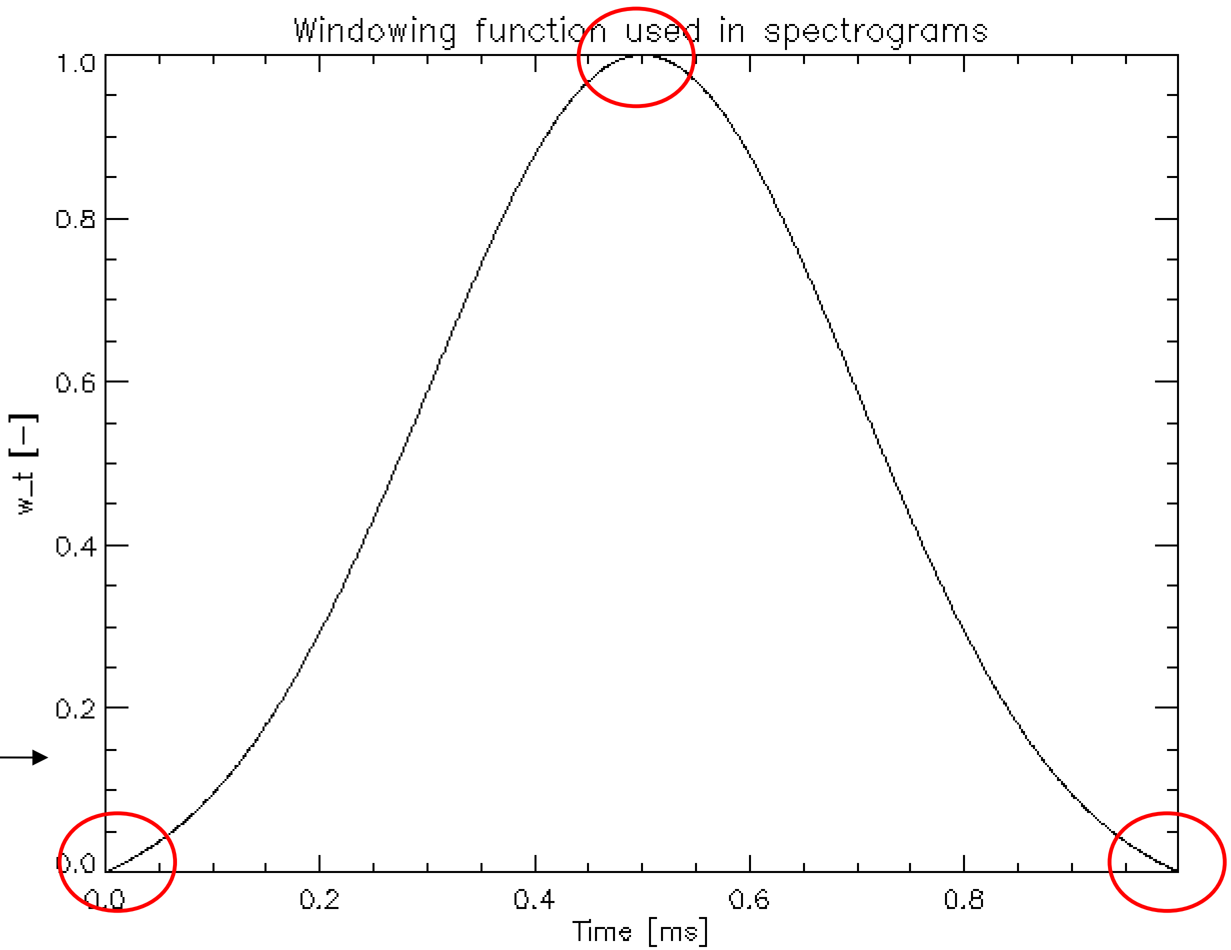
- Multiplication of signal by an appropriate function
- Most common:

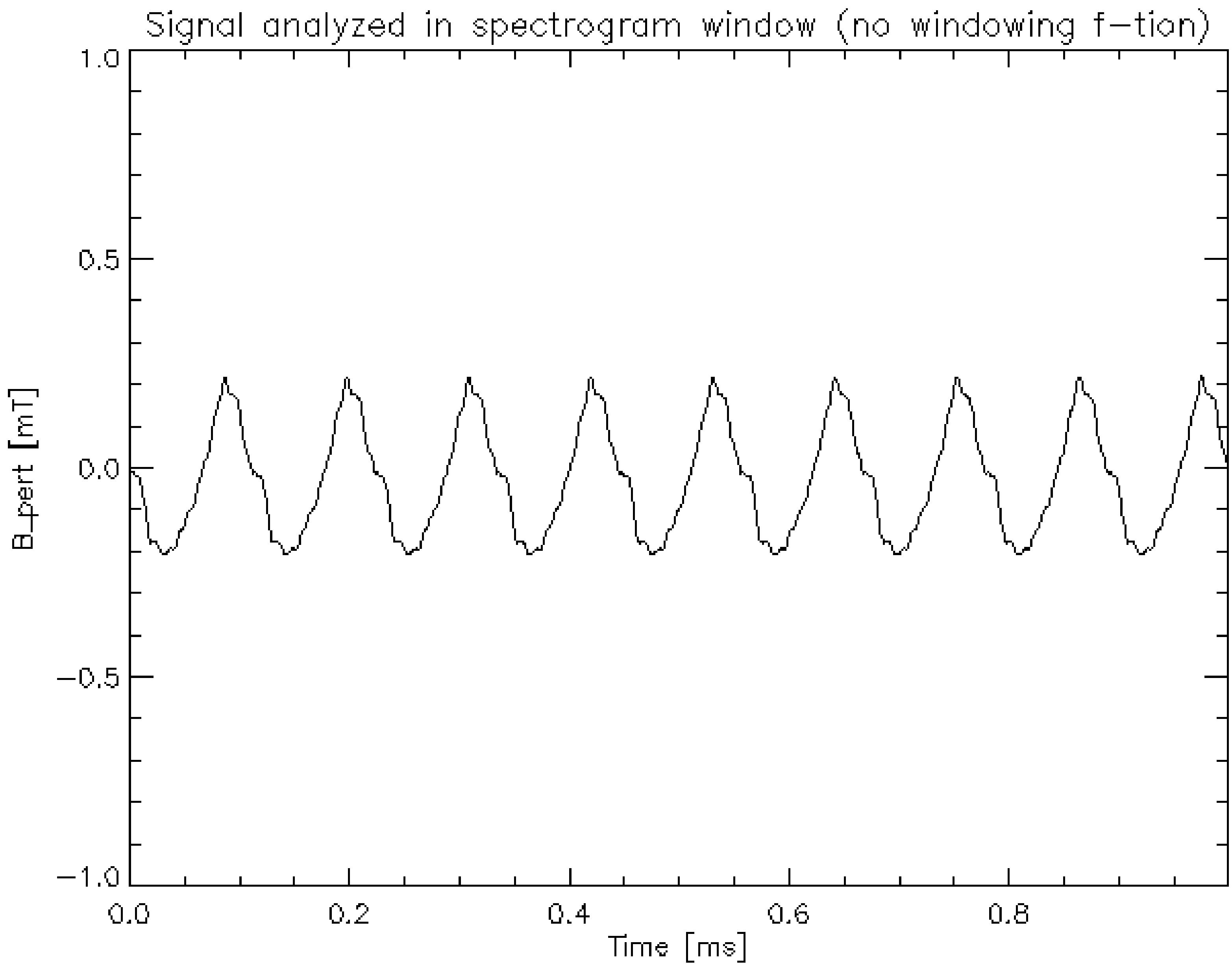
Hamming,

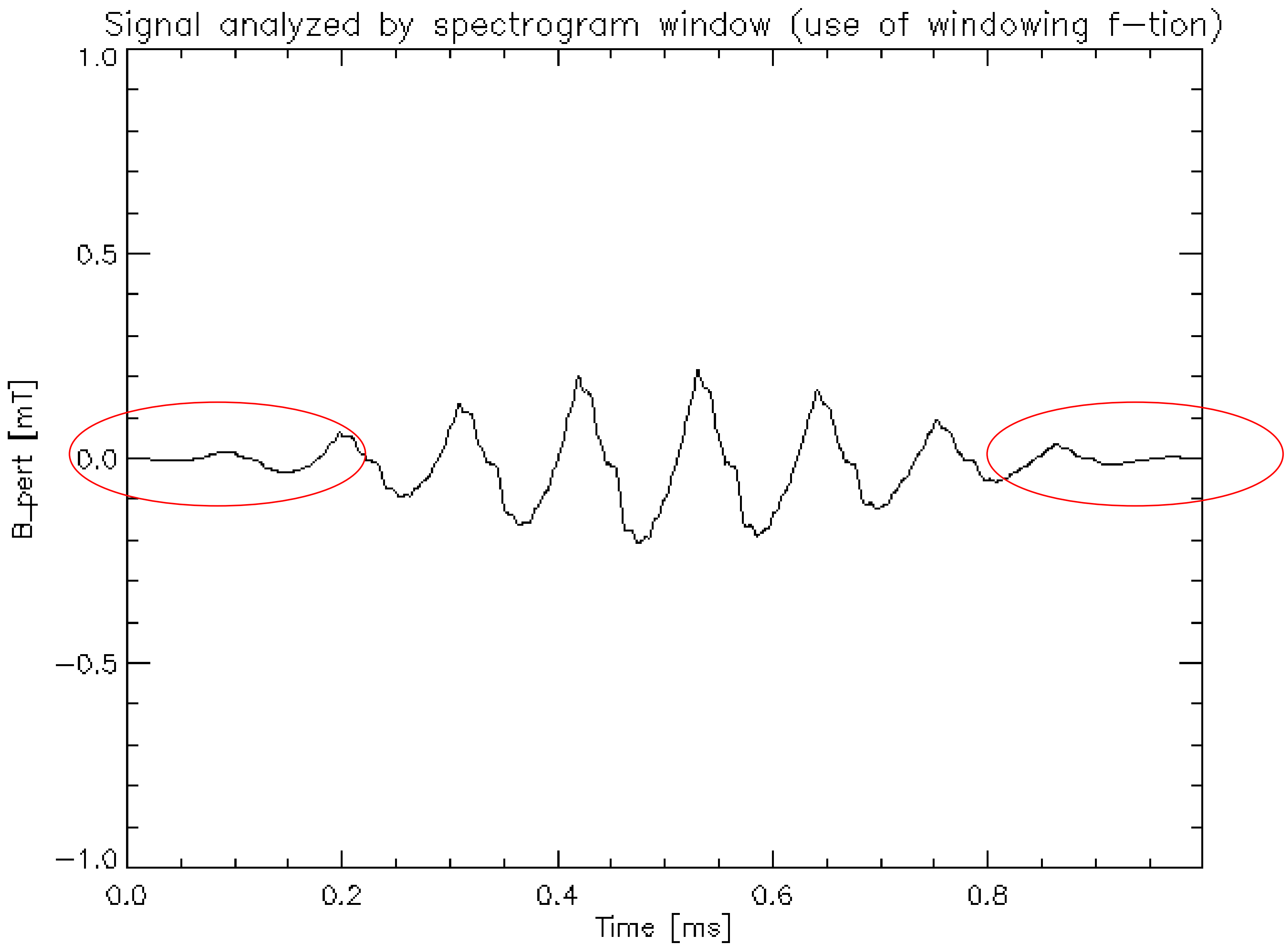
Gaussian, …

All have 1 in center and 0 at edges

This is my favorite one →




Signal analyzed in spectrogram window (no windowing f-tion)
1.0
0.5
0.0
-0.5
-1.0
B_pert [mT]
0.0
0.2
0.4
0.6
0.8
Time [ms]



Signal analyzed by spectrogram window (use of windowing f–tion)
1.0
0.5
0.0
–0.5
–1.0
B_pert [mT]
0.0
0.2
0.4
0.6
0.8
Time [ms]

FFT algorithm

- Finally, it is safe to apply FFT
- It's IDL implementation:

$$F(\nu) = \frac{1}{N} \sum_{x=0}^{N-1} f(x) \mathrm{e}^{-\mathrm{i} 2\pi \nu x / N}$$

- Most commonly used in temporal domain of signal – to obtain <u>frequencies</u> of signal
  - However, application of FFT on spatial domain may yield *m* mode number of island

Output of FFT (IDL)

- Input – array of time evolution of signal
- Output:
  - Array of same size as input
  - Complex numbers representing Fourier coefficients
  - Each coefficient – "strength" of given frequency in signal
  - Frequencies go from -*f_sample*/2 to +*f_sample*/2
  - First half of output array – positive frequencies, second half – negative frequencies
  - Both halves are the same in absolute values

How to process FFT output

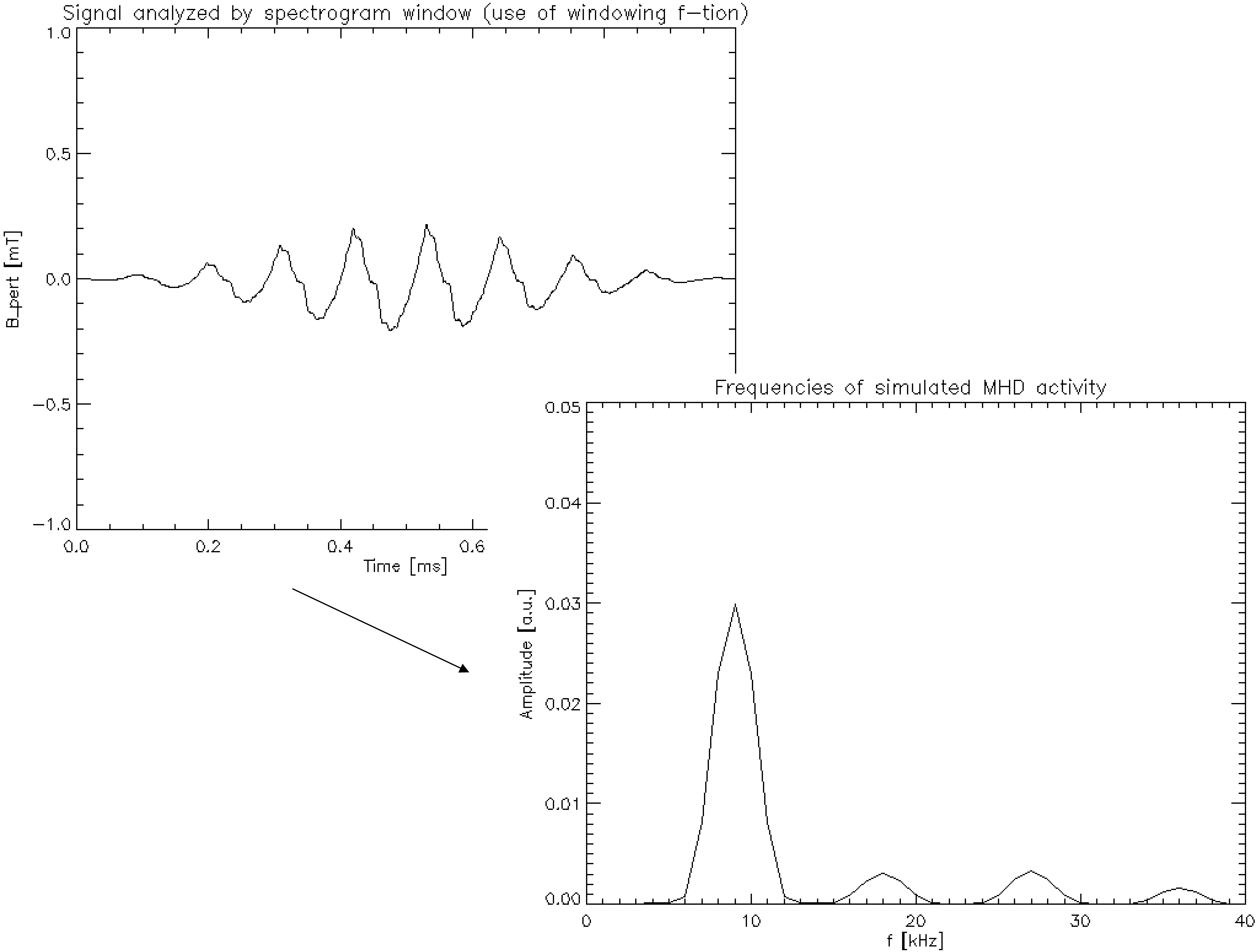
1. Take absolute value of output (the one from complex analysis) to obtain magnitude
   - In the case of interest – Daniel sent you paper when phase is used instead of magnitude
2. Take only first half of signal – the second is the same as the first
3. Calculate which magnitude data point represents which frequency:

$$f(i) = \frac{f_{sample}}{2} \frac{i}{N_{win}}$$

Frequencies in FFT

$$f(i) = \frac{f_{sample}}{2} \frac{i}{N_{win}}$$

- *i* – index of output data point, *N_win* – number of data points in window
- *f_sample*/2 also called Nyquist frequency
  - If you want detection of higher frequencies – you need to increase sampling frequency
- If you want good frequency resolution – you need wider signal window
  - With stationary phenomena – just measure longer
  - In plasma difficult – phenomena last only some limited time – we cannot do much about it



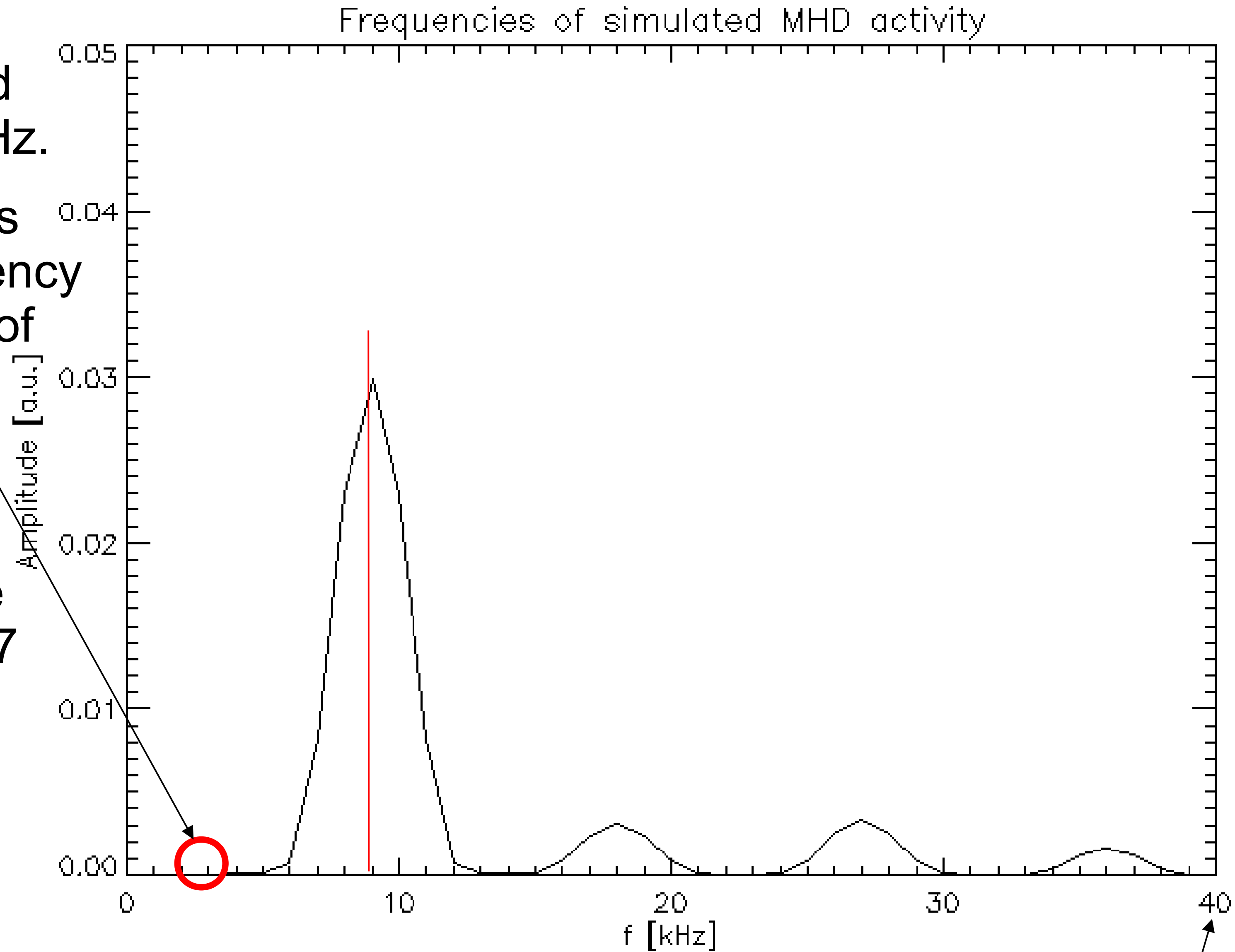
Signal analyzed by spectrogram window (use of windowing f–tion)
B_pert [mT]
Time [ms]
1.0
0.5
0.0
–0.5
–1.0
0.0
0.2
0.4
0.6
Frequencies of simulated MHD activity
Amplitude [a.u.]
f [kHz]
0.05
0.04
0.03
0.02
0.01
0.00
0
10
20
30
40

Interpretation of FFT

We had m = 3 island rotating with f = 3 kHz.

However, FFT shows that dominant frequency is at 9 kHz. No sign of 3 kHz anywhere.

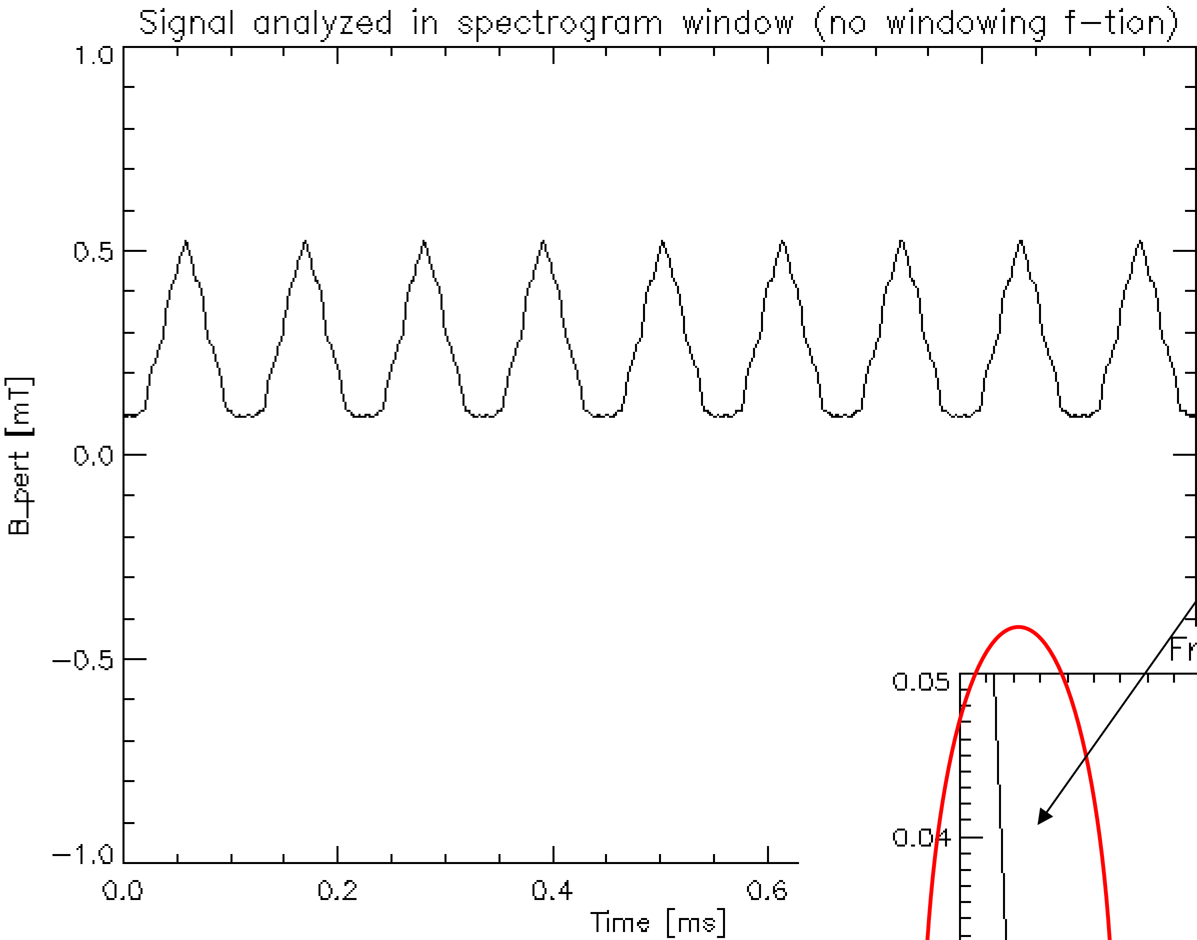
There are also some small peaks at 18, 27 And 36 kHz.



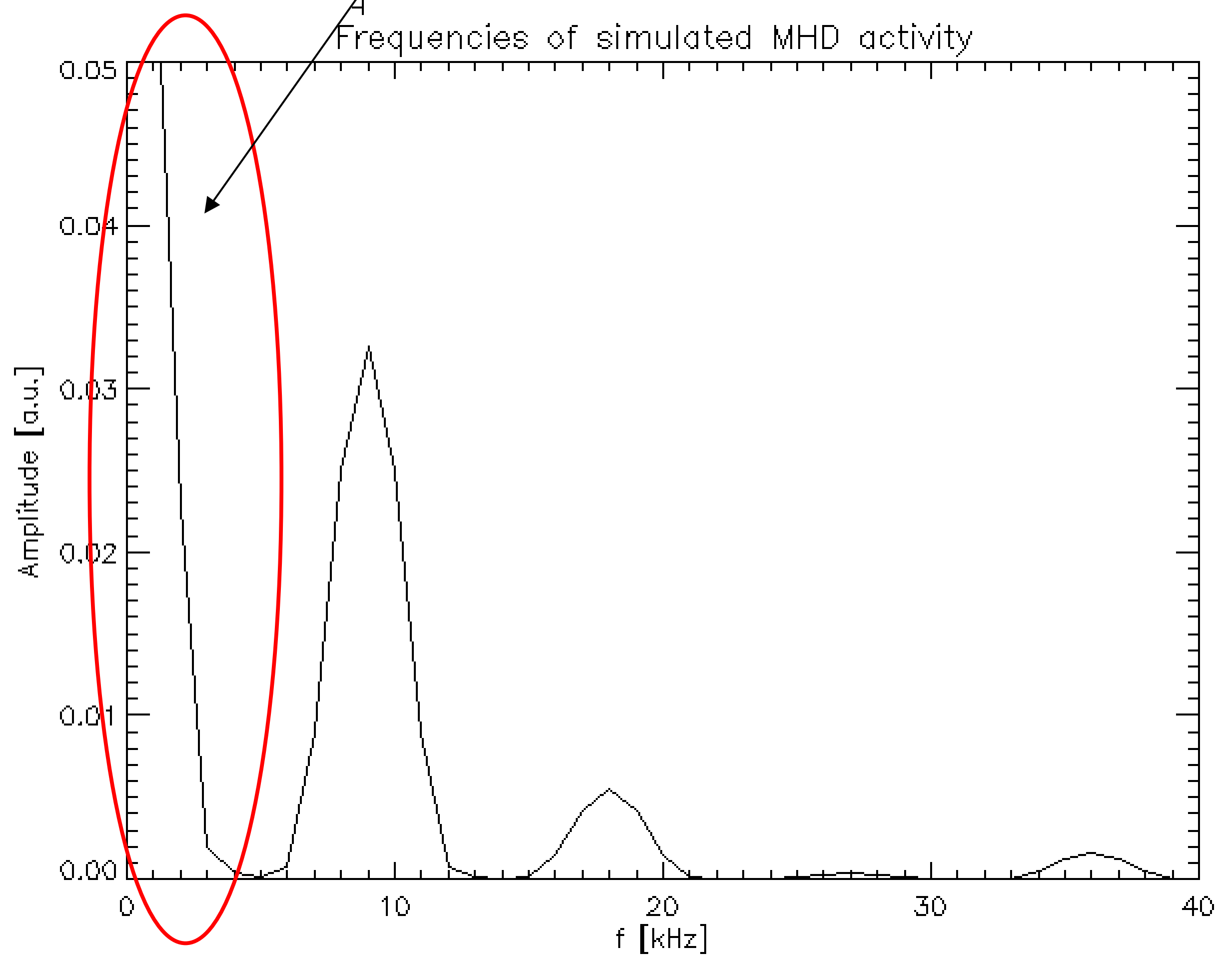
Result naturally goes up to 500 kHz, but there was nothing from 50 kHz higher, so I just cut it here

Interpretation of FFT

- 18, 27 and 36 kHz part of result are just higher harmonics of main result of 9 kHz
- Why is there 9 kHz instead of 3 kHz?
  - Because *m* = 3, there are 3 same structures rotating at the same time
  - Therefore it seems that rotation is 3 times faster than it actually is
  - Thus, in order to successfully identify frequency of island rotation, it is necessary to know *m* first!



By the way, this is what you get if you have DC offset in signal. Please apply windowing and DC elimination properly...

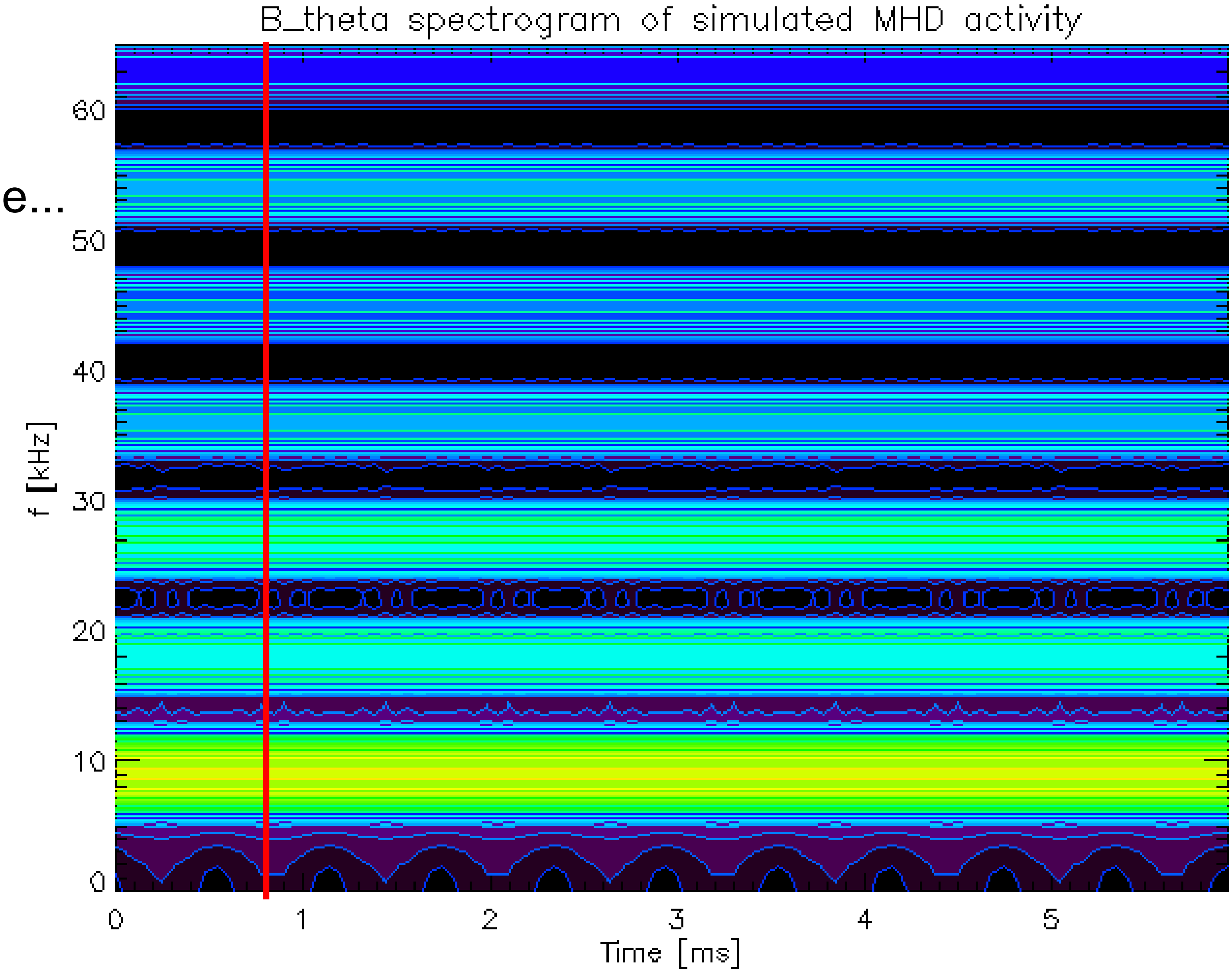


Spectrograms

- Dividing signal into many time windows and doing FFT in each of them separately
- This way – each time window represents different time interval in data
- Useful to monitor how frequencies in signal change over time
  - To identify time of island existence

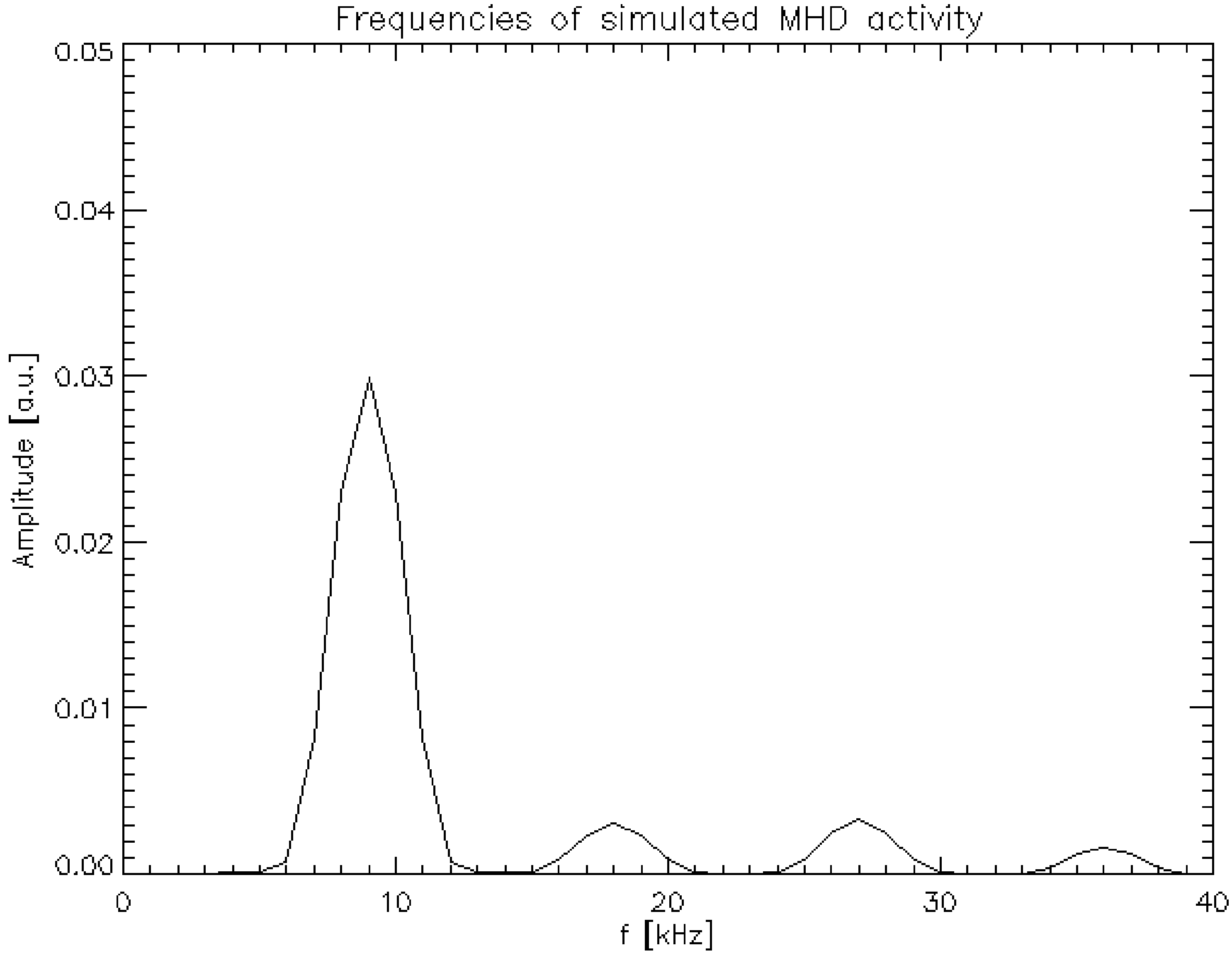
Model – constant frequency in time

On the red line...



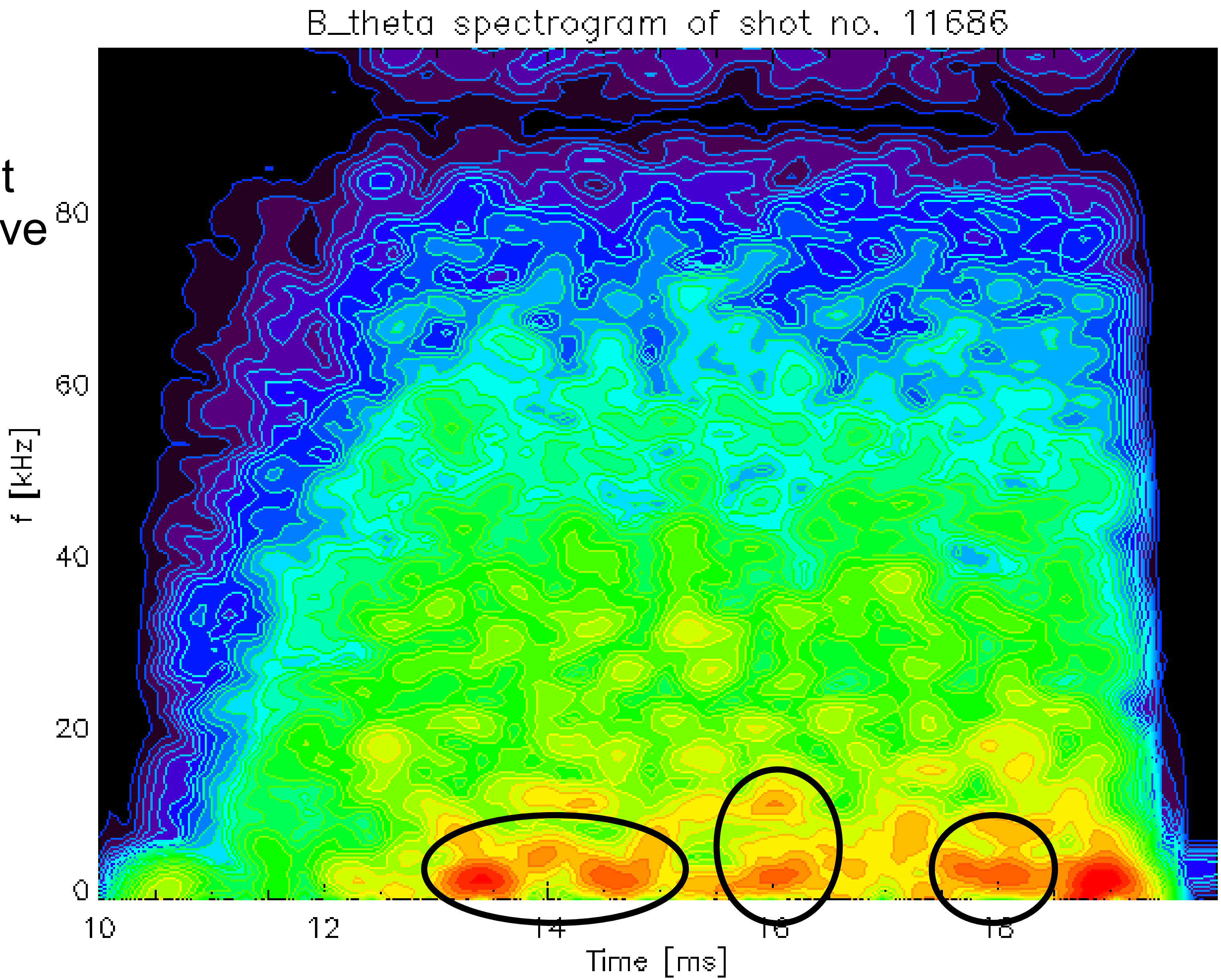
Model – constant frequency in time

There is just simple FFT transform...



Application to experimental data

It is evident that phenomena have <u>finite duration</u>



Spectrogram dilemma

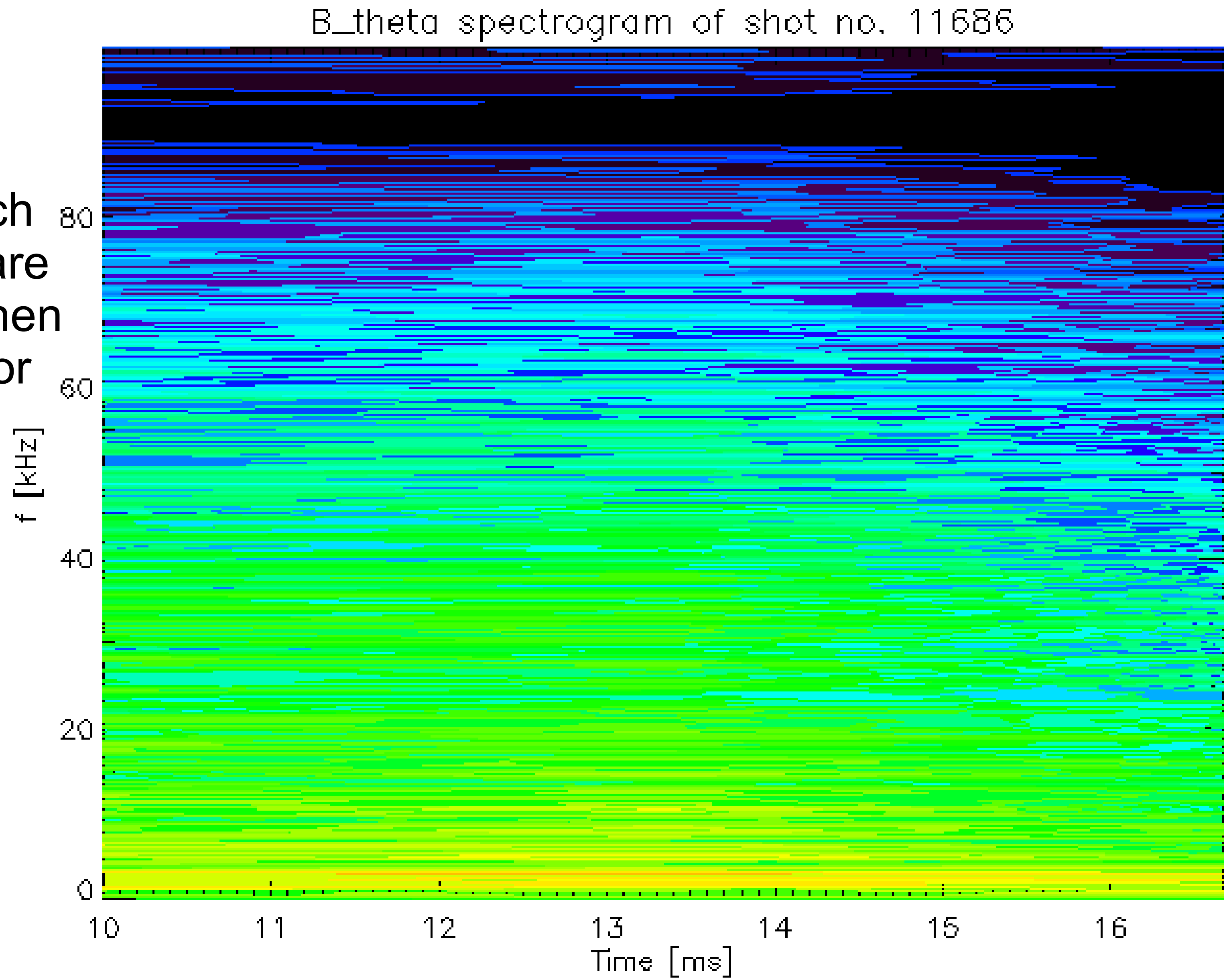
- To capture phenomena of short existence in spectrogram, small windows for FFT are necessary
- However, frequency resolution is given by:

$$f(i) = \frac{f_{sample}}{2} \frac{i}{N_{win}}$$

- Narrow window – bad frequency resolution
- Good spectrogram – <u>trade-off between good time resolution and good frequency resolution</u>
- Making windows <u>overlap helps a lot</u>

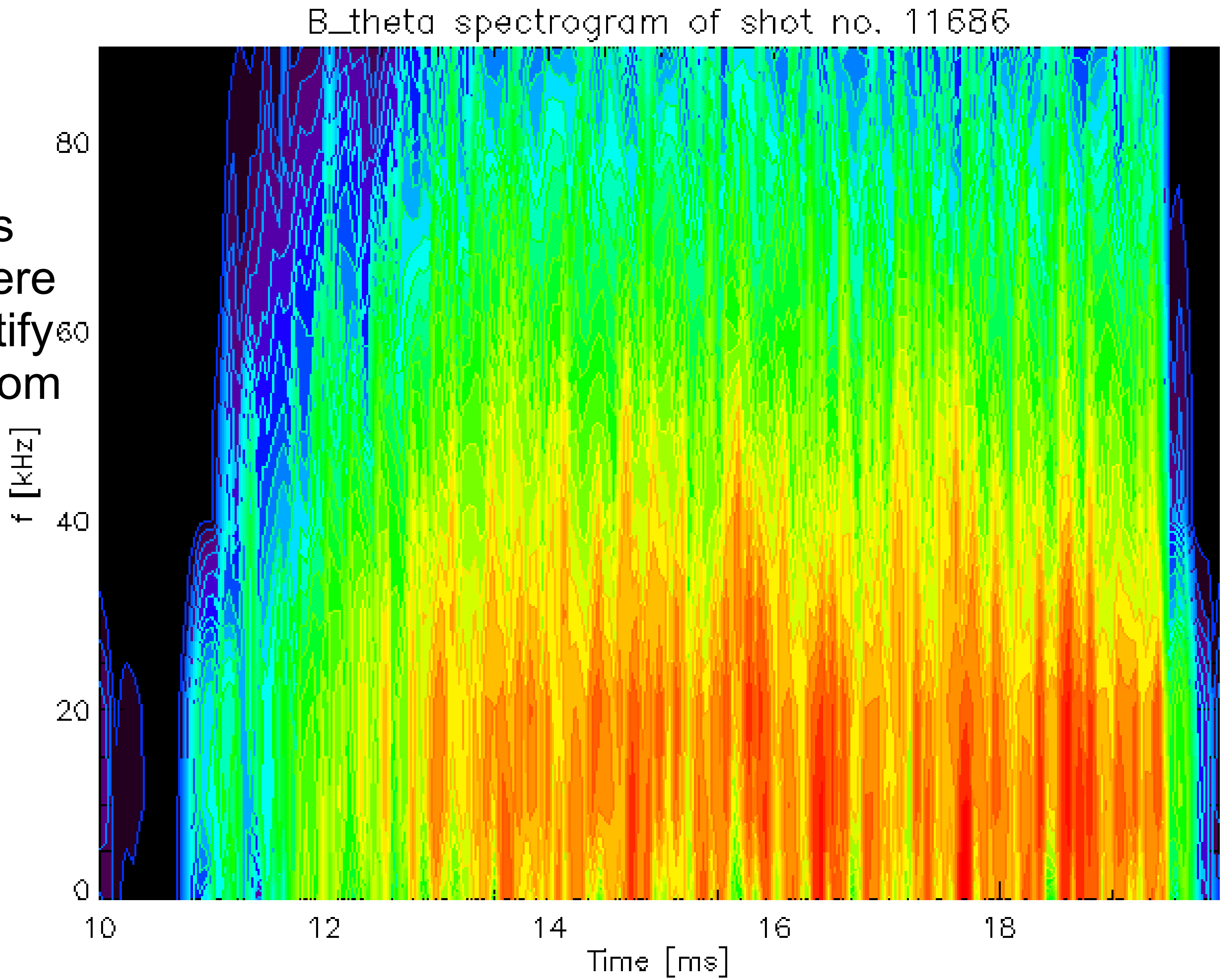
When window is too wide

We can see each frequency, but are unable to tell when did they occur, or to distinguish islands from each other



When window is too narrow

We can identify the time of events in plasma, but there is no way to identify their frequency from this mess



Correlation analysis

- As useful in data processing as FFT
- Commonly used in both temporal and spatial domain
- Many interpretations on actual meaning of result
  - So we will only discuss the basic algorithm and what it does to known data provided by island simulation

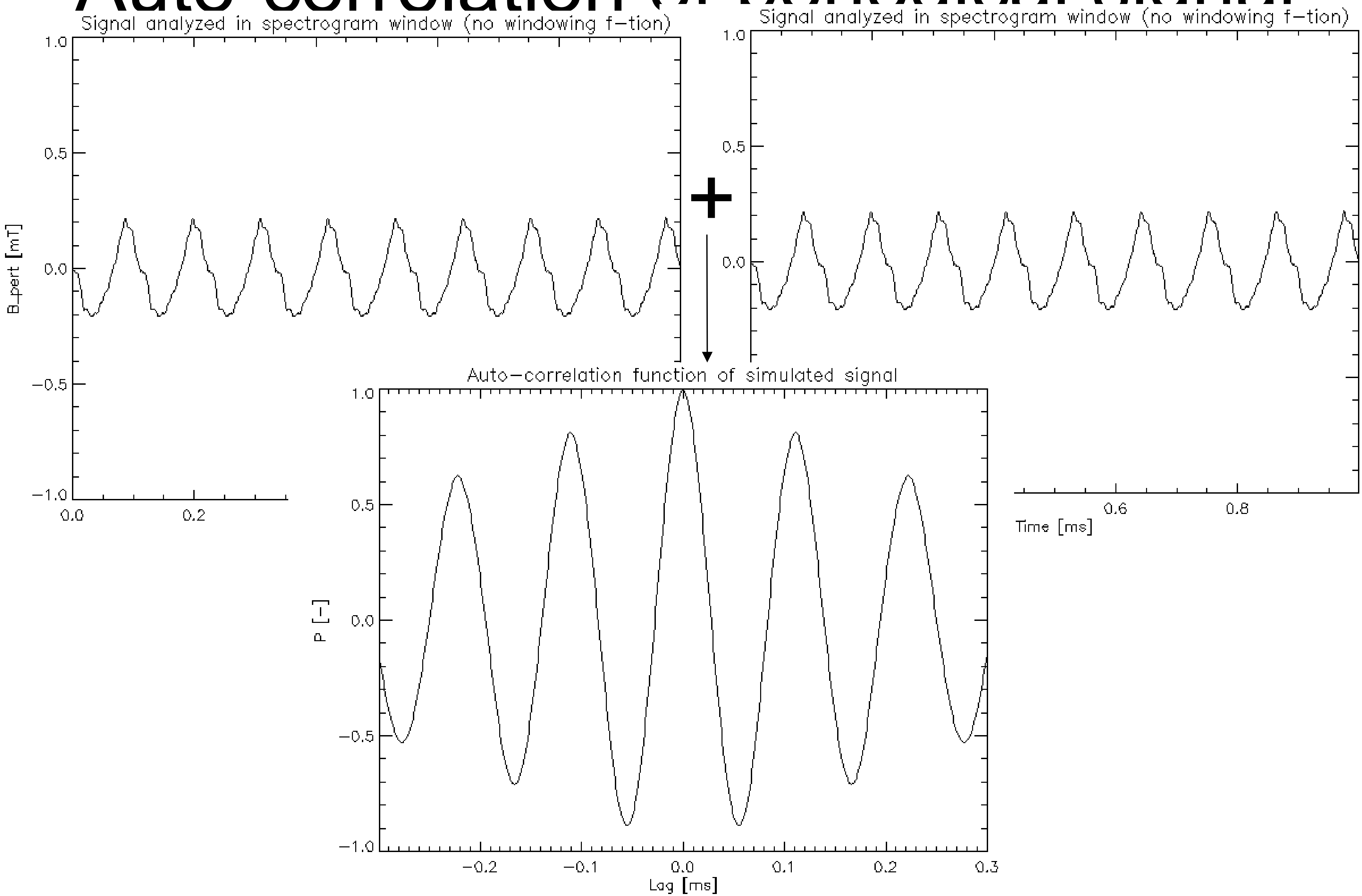
C_correlate (IDL implementation)

$$P_{xy}(L) = \frac{\sum_{k=0}^{N-|L|-1} (x_{k+|L|} - \bar{x}) \cdot (y_k - \bar{y})}{\sqrt{\left[\sum_{k=0}^{N-1} (x_k - \bar{x})^2\right] \cdot \left[\sum_{k=0}^{N-1} (y_k - \bar{y})^2\right]}} \quad \text{for } L < 0$$

$$P_{xy}(L) = \frac{\sum_{k=0}^{N-L-1} (x_k - \bar{x}) \cdot (y_{k+L} - \bar{y})}{\sqrt{\left[\sum_{k=0}^{N-1} (x_k - \bar{x})^2\right] \cdot \left[\sum_{k=0}^{N-1} (y_k - \bar{y})^2\right]}} \quad \text{for } L > 0$$

- *x* and *y* represent signals with N data points
- *L* has dimension of data point index and $t_L = \frac{L}{f_{sample}}$
- Barred x and y represent averages
- Therefore, denominator is geometrical average of signal variances – this causes that *P* is from (-1,1)

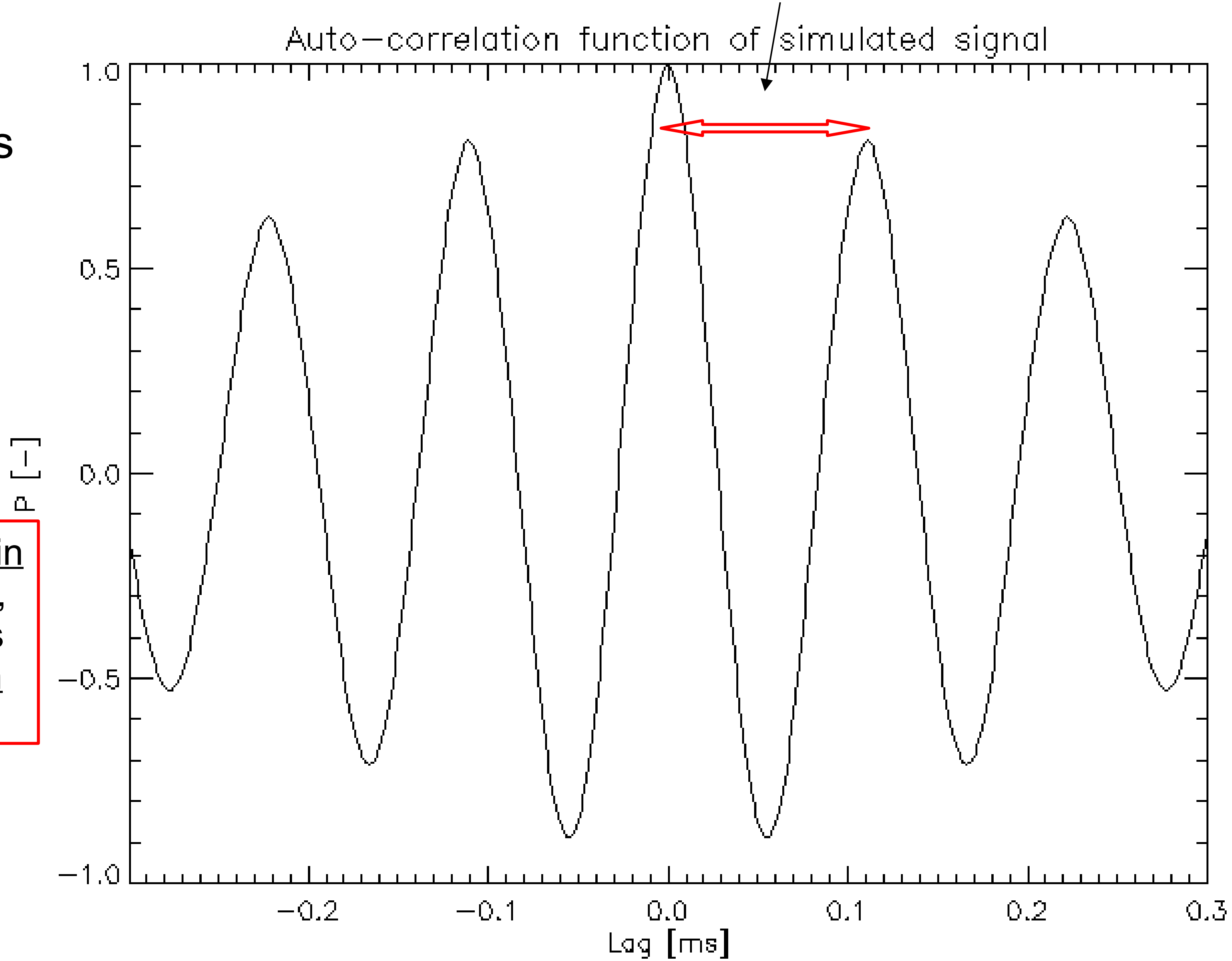
Auto-correlation of periodical signal



This is how periodic signal "interacts" with itself

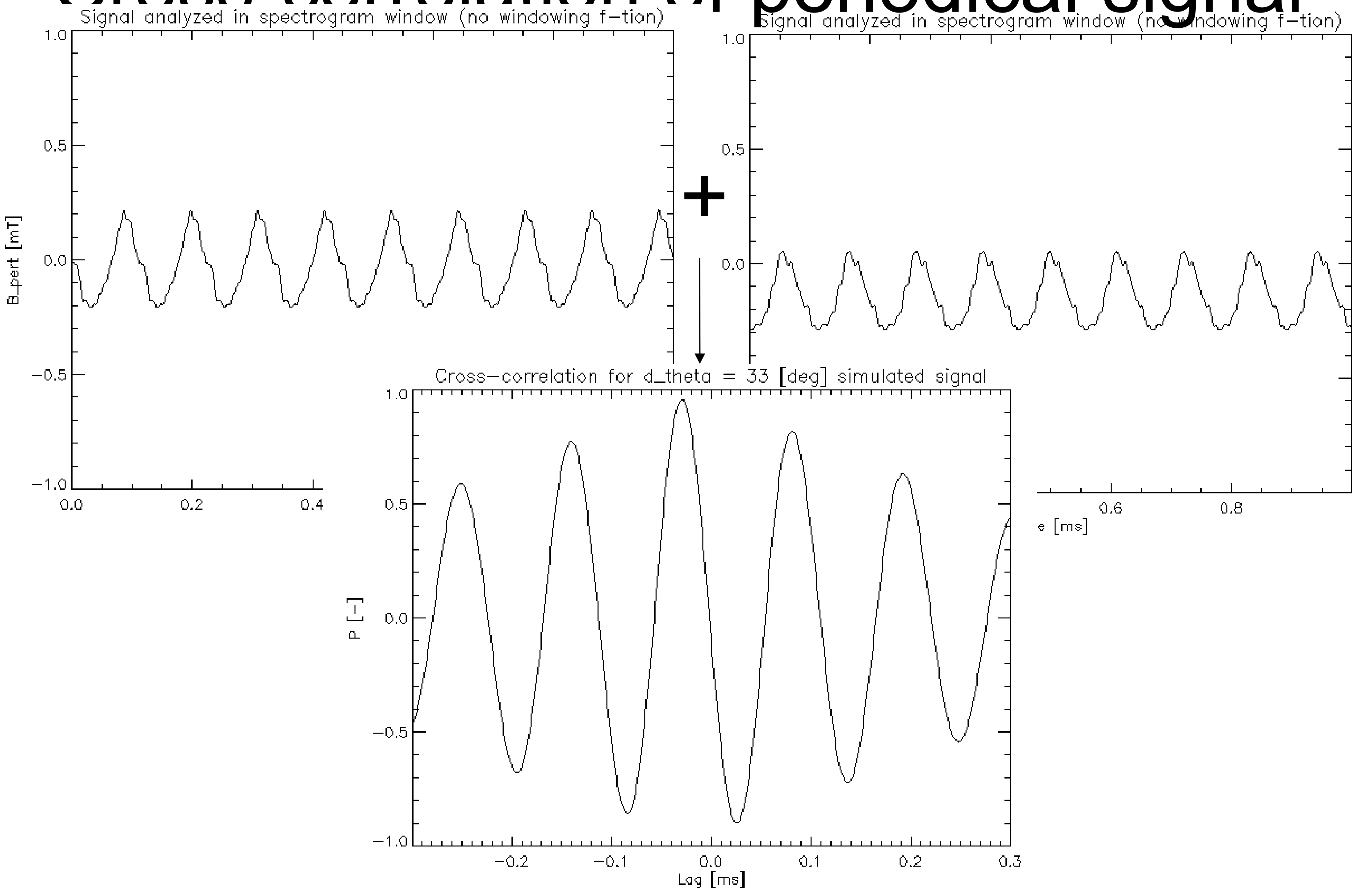
This distance of maxima implies *f* = 9.26 kHz

Output is always normalized to (-1,1) interval



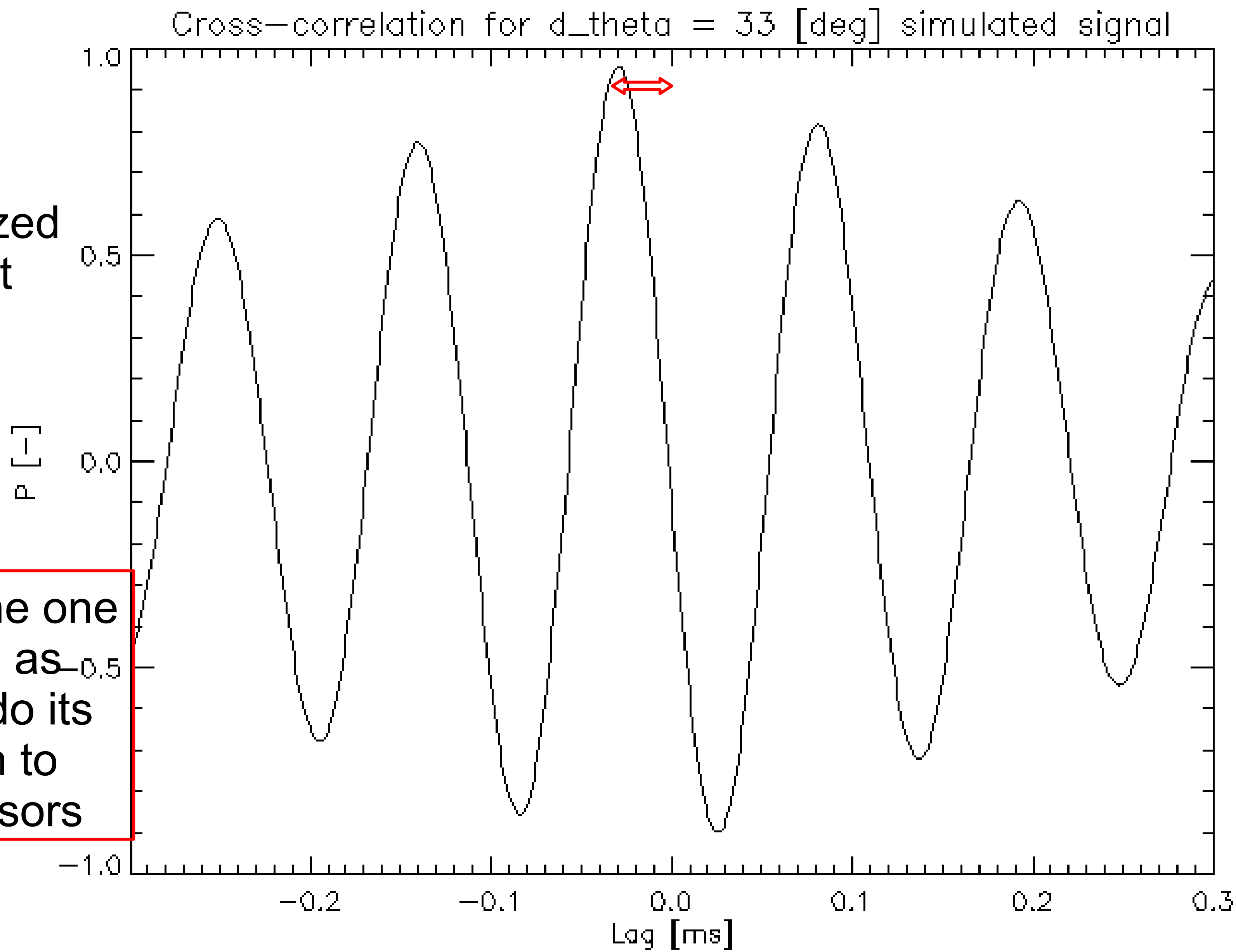
<u>On temporal domain</u> for periodic signals, correlation analysis <u>is less reliable than FFT</u>

Cross-correlation of periodical signal



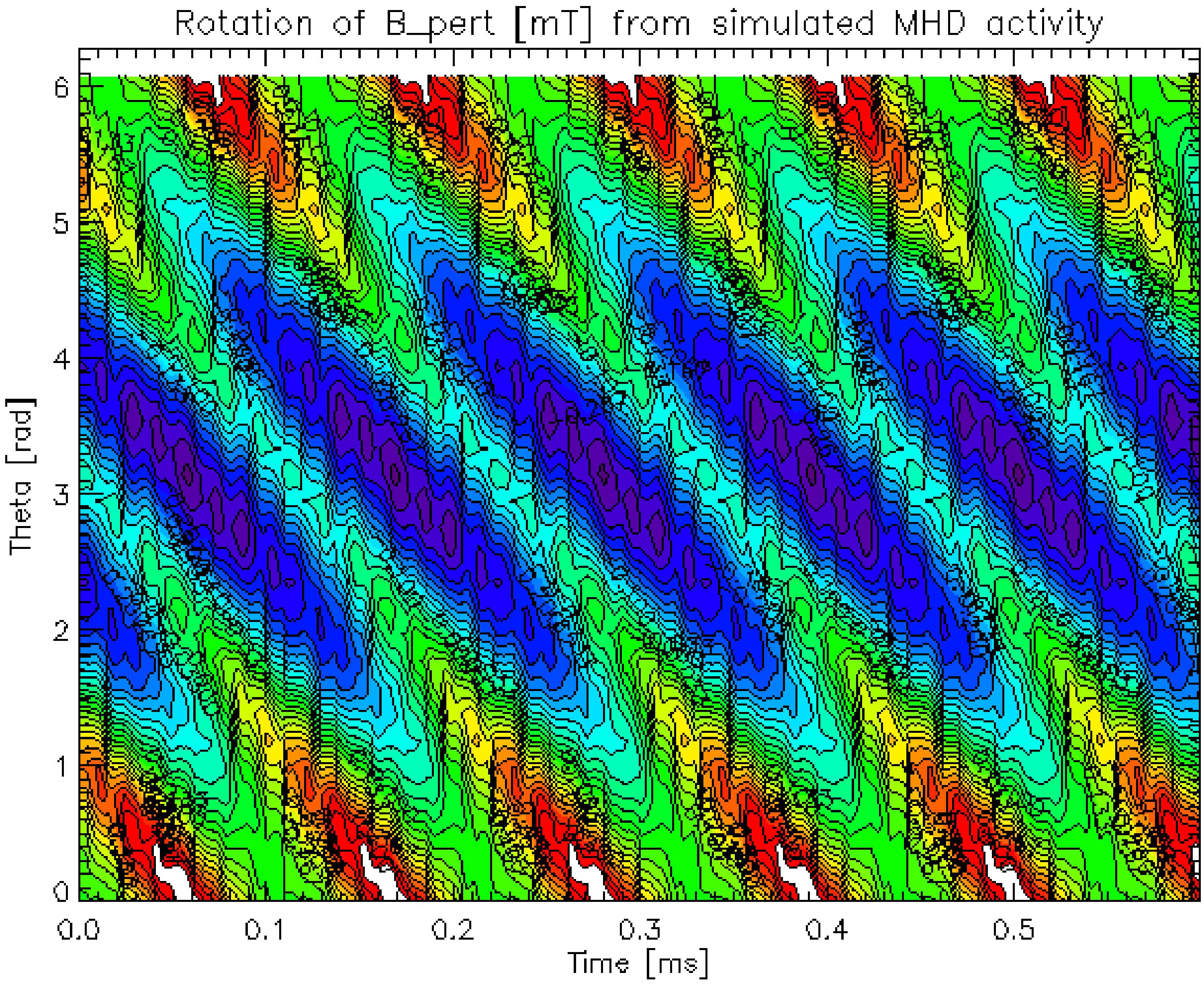
Due to phase shift of signals, maximum of cross correlation is not at lag = 0

It is still normalized to (-1,1), thought



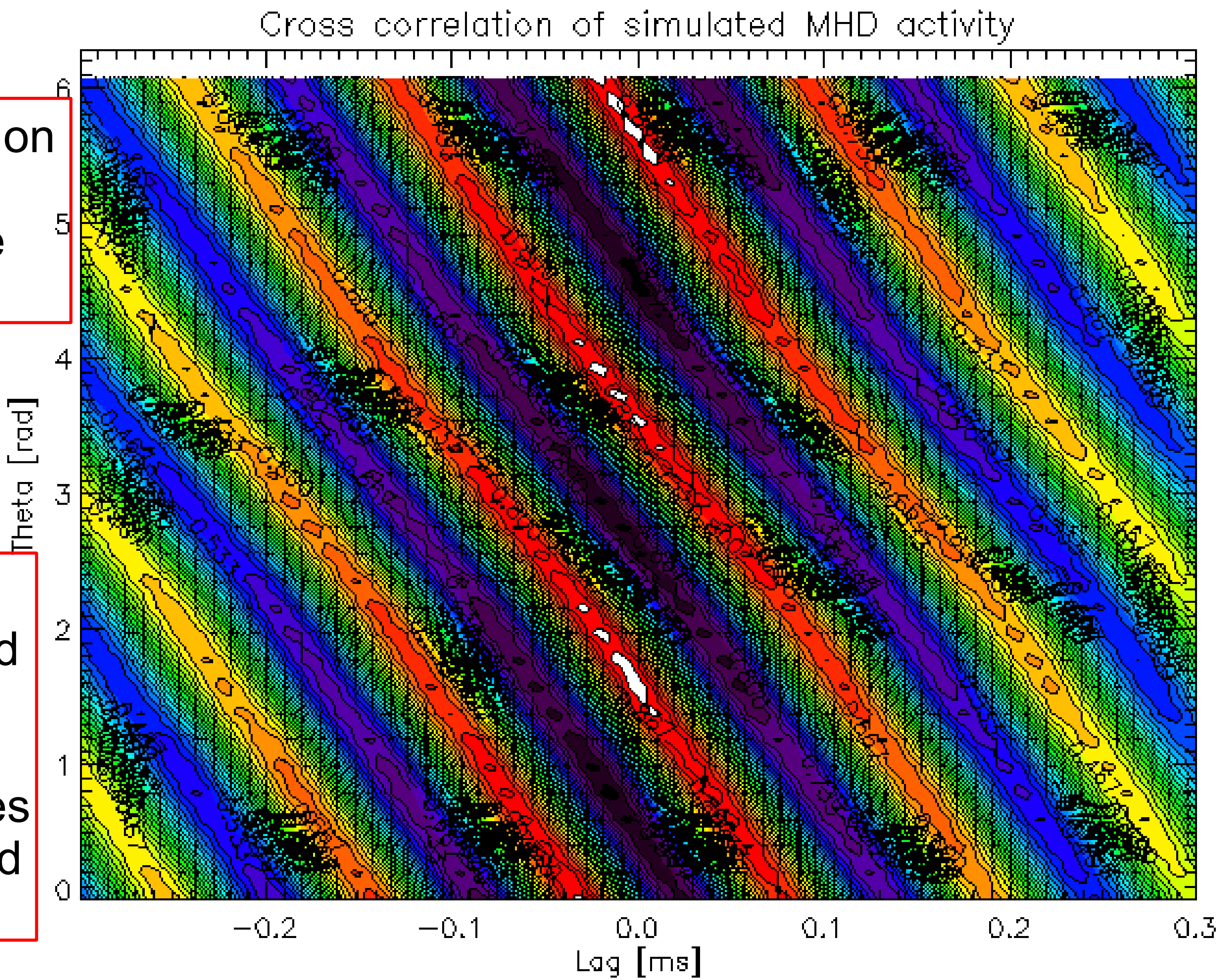
Let us now define one sensor (angle θ) as Reference and do its cross correlation to all the other sensors

Original time-space signal:



Upon cross-correlation:

Inherent normalization of cross-correlation removed magnitude differences

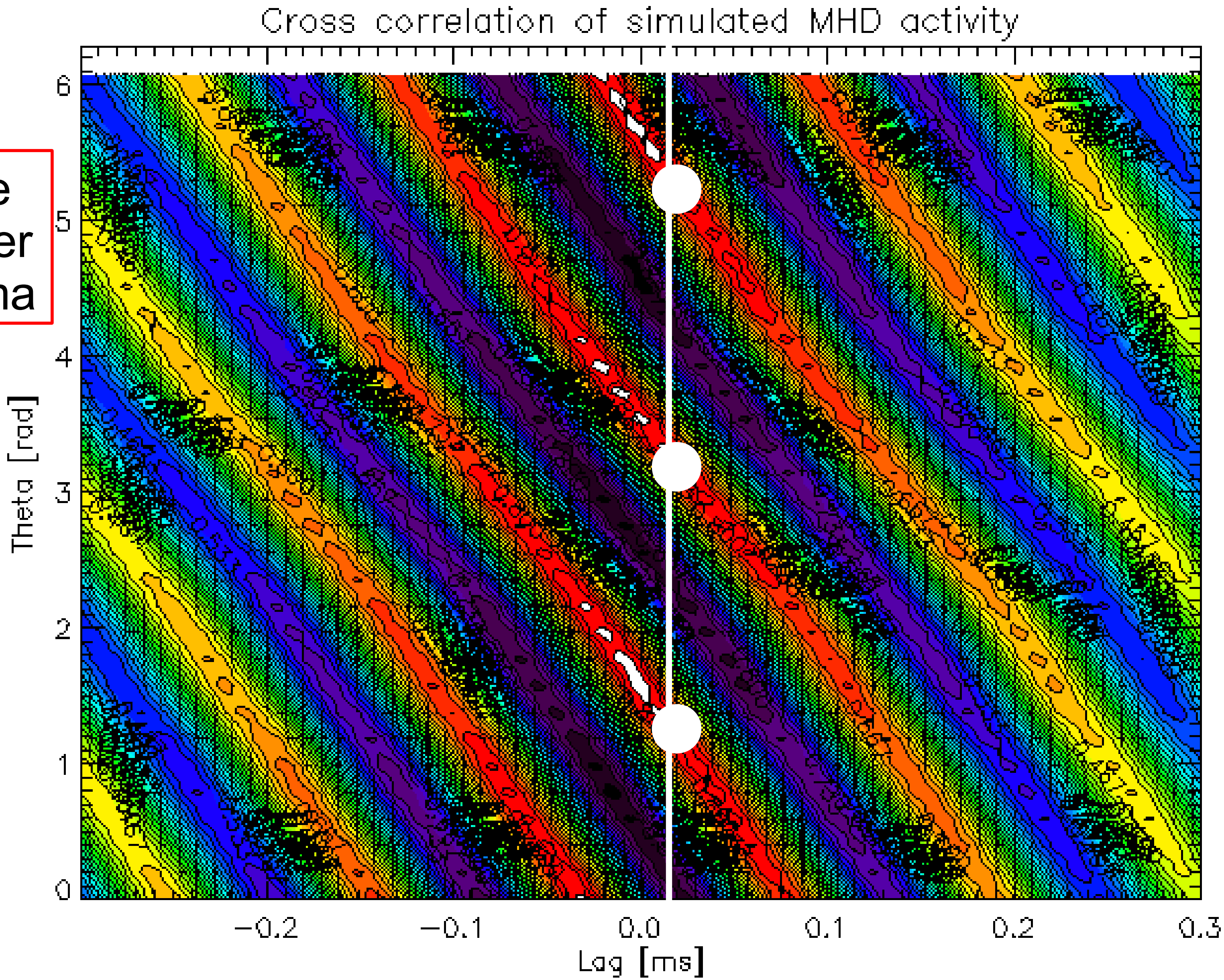


Periodical character of data was amplified

Algorithm ignores signal shape – it sees only its repetition and similarity

Identification of *m* mode number

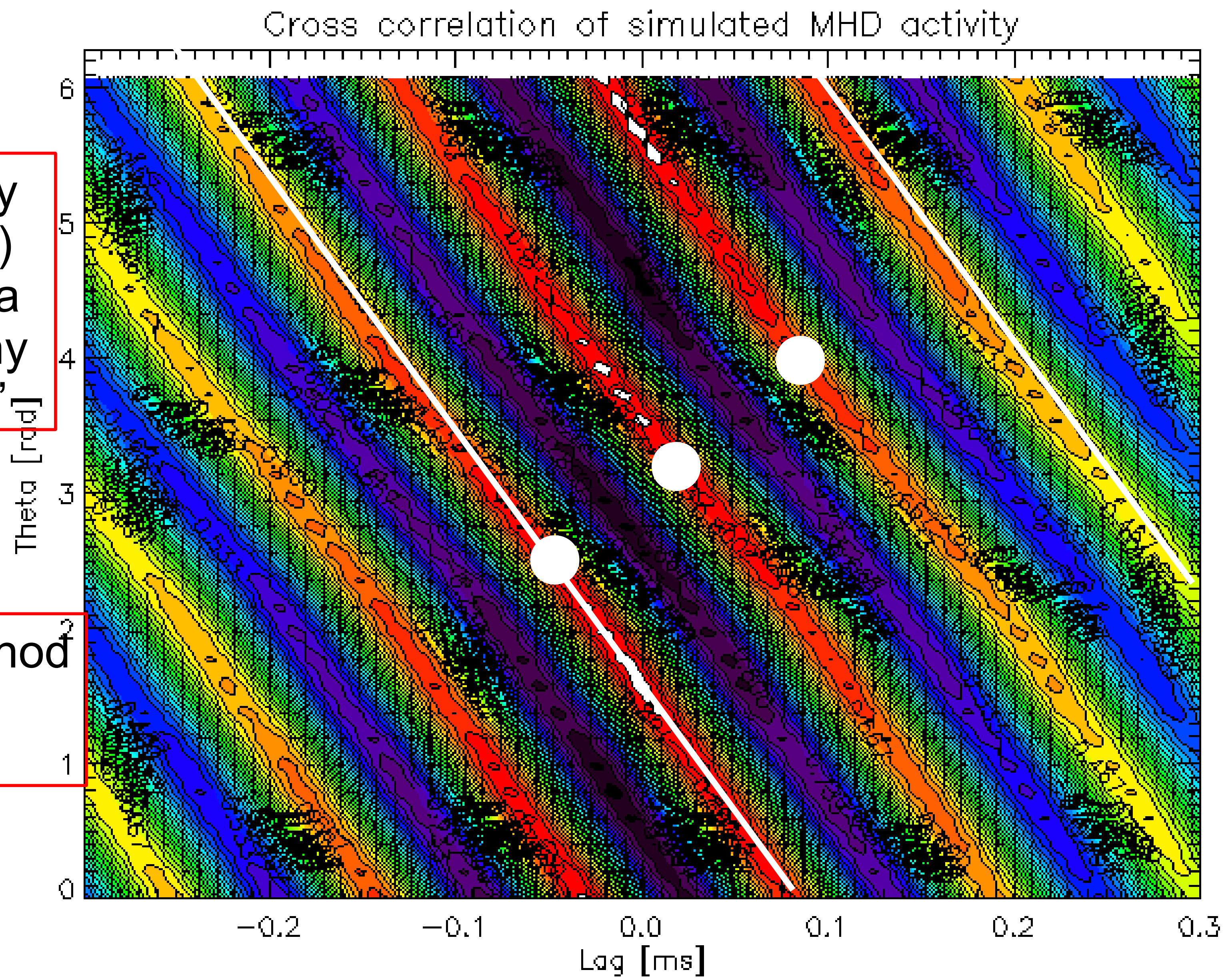
Drawing vertical line
and counting number
Of maxima or minima



Identification of *m* mode number

Following periodicity of a field line (white) using signal maxima and count how many maxima are “inside”

Recommended method if there are missing coils!

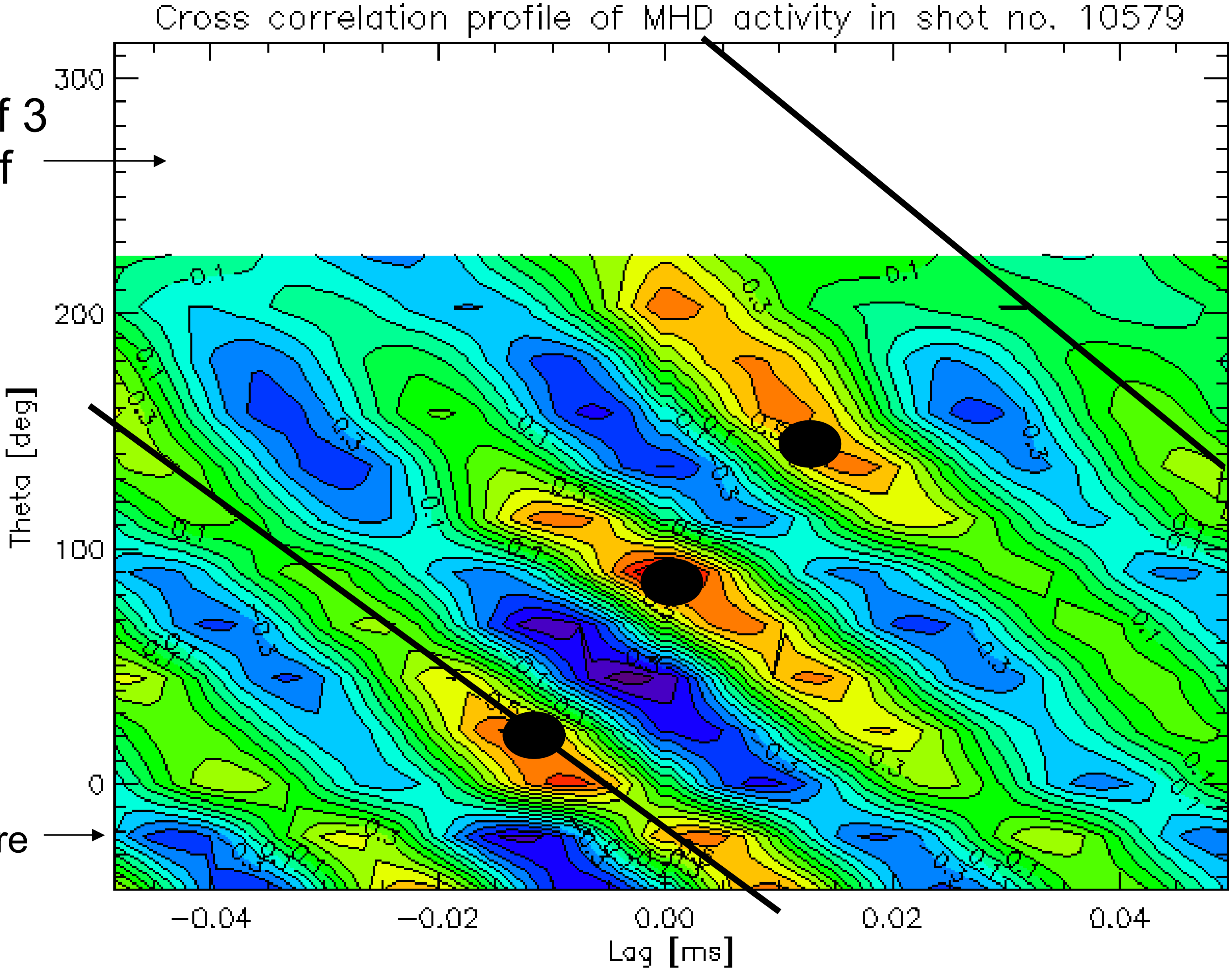


Application to experimental data

Notice removal of 3 coils on bottom of array

q_edge = 4.5, so this is deep in plasma

Coil 15 and 16 are here



Summary

- Both FFT and c_corr can be used on temporal and spatial domains to extract island frequency and structure
- FFT
  - Necessary to slightly modify the signal before use
  - Better for time domain, especially to detect changes of frequencies with time
  - To be used on spatial domain, it would be necessary to have more coils or to do reliable interpolation
  - You are encouraged to try this

Summary

- Cross-correlation
  - Excellent for island tracking – normalizes the signals, inhibits fluctuations and brings forward its periodical character
  - Most reliable method for *m* extraction from data
  - However it is not as reliable on temporal domain as FFT