DIRECT DETERMINATION OF THE ELECTRON ENERGY CONFINEMENT TIME OF A TOKAMAK PLASMA*

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A method of determining the electron energy confinement time τ_E is presented which does not require the absolute electron temperatures and densities, or their spatial profiles. A short heating pulse is applied to the plasma; the subsequent decay of electron temperature yields τ_E.

The electron energy confinement time τ_E is an important parameter for assessing the effectiveness of the magnetic confinement of plasmas in tokamak devices. This quantity is defined by the energy balance equation [1]

$$\mathrm{d}U_e/\mathrm{d}t = -U_e/\tau_E + S(t), \tag{1}$$

where

$$U_e = 2\pi R \int_0^a (3/2)\, n_e T_e\, 2\pi r \,\mathrm{d}r, \tag{2}$$

is the total electron thermal energy contained within a torus of minor radius a and major radius R. T_e and n_e are the (spatially varying) electron temperature and density, respectively. The source function

$$S(t) = (V - L\ \mathrm{d}I/\mathrm{d}t)\, I \tag{3}$$

represents the joule heating by the current I driven through the plasma. V is the loop voltage (the electric field integrated over $2\pi R$) and L is the inductance of the plasma current loop.

In conventional measurements [e.g., 2] of τ_E it is assumed that the plasma is in near steady state with $\mathrm{d}U_e/\mathrm{d}t = \mathrm{d}I/\mathrm{d}t \approx 0$, in which case $\tau_E = U_e/VI$. This equation is the basis of all determinations of τ_E made hitherto. It may be seen that the method requires not only the values of T_e and n_e, but also a knowledge of their spatial profiles. Such numbers are not easily found, particularly in tokamaks with noncircular cross sections [3] or in stellarators [4] in which the plasma quantities have complicated distributions. Also, the assumption that $\mathrm{d}U_e/\mathrm{d}t \approx 0$ is often not a good one, especially in smaller tokamaks where the duration of the discharge τ_D is relatively short.

In this letter we describe a different method of determining τ_E which does not require knowledge of the absolute values of T_e and n_e and which is also quite insensitive to the spatial profiles of these quantities. A perturbing heating pulse of short duration ($\ll \tau_D$) is applied to the plasma, thus causing a small increase ΔU_e in the kinetic energy of the electrons. After the heating pulse one observes the temporal decay of the energy increment $\Delta U_e(t)$. Similar concepts have been used for many years in thermal conductivity measurements [5,6], but have not yet been applied to tokamak discharges.

This method, like the conventional one, is based on eq. (1). One compares the equation for two plasma discharges, one with and one without the supplemental heating pulse, and by subtracting one from the other, obtains an equation for the increment ΔU_e. Note that this comparison is made *after* termination of the pulse. Moreover, since the pulse is taken to be too weak to change the transport properties of the plasma significantly, it follows that n_e, τ_E and the joule heating term S are unaffected by the presence or absence of the pulse. Hence, ΔU_e is proportional to $\langle \Delta T_e \rangle$ (spatially averaged temperature) and

$$\frac{\mathrm{d}}{\mathrm{d}t}\Delta U_e = -\frac{\Delta U_e}{\tau_E}, \tag{4}$$

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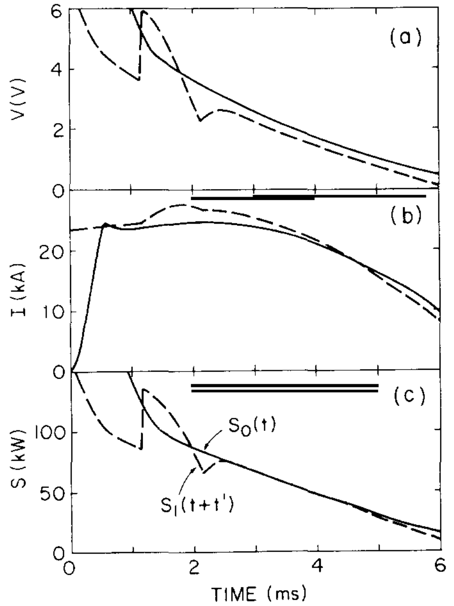

Fig. 1. (a) Loop voltage, (b) plasma current, and (c) ohmic power dissipation for plasmas without a heating pulse (solid curves) and with a heating pulse (dashed curves).

which for τ_E independent of t, yields the exponential behavior, $\Delta U_e = \Delta U_{e0} \exp(-t/\tau_E)$. This, then, is the basis of our method of finding τ_E on the Versator I tokamak. It is seen to be independent of the values of T_e and n_e or their spatial variation, as long as the heating pulse does not alter the shape of the discharge or its transport characteristics.

Versator I is a research tokamak [7] of minor radius 14 cm and major radius 54 cm, operating in a toroidal magnetic field of ~ 5 kG, and generating plasma currents of ~ 30 kA. Power is supplied by capacitor banks. The solid lines of fig. 1 (a) and (b) illustrate typical time histories of the loop voltage $V(t)$ and plasma current $I(t)$. The spatially averaged electron temperature $\langle T_e \rangle$ is approximately 120 eV, a value it reaches about 4 ms after initiation of the discharge. The spatially averaged electron density $\langle n_e \rangle \approx 2 \times 10^{13}$ cm^{-3}. The minor radius of the hot plasma column is ~ 12 cm. The plasma is hydrogen with $T_i \approx 50$ eV; the effective Z lies between 2 and 3.

The heating pulse of about 1 ms duration is applied to the plasma by discharging a supplementary capacitor bank into the ohmic heating transformer. The curves of figs. 1 (a) and (b) show the effect of the pulse on the voltage and current characteristics of a typical plasma. Larger and smaller pulses cause similar effects, with changes mainly in the amplitude of the response. For a perturbation analysis, it is desirable to study the smallest possible pulse which produces a measurable change. Because of our present crude temperature measurements (described below), temperature changes $\Delta T_e/T_e$ of about 10% are needed. The heating pulse in fig. 1 is the smallest pulse which causes changes of this magnitude, and consequently it is used in the analysis presented below.

The derivation of eq. (4) required that the ohmic dissipation term $S(t)$ be unaffected by the pulsed heating perturbation. To verify this fact, we used eq. (3) and the data of fig. 1 to compute [‡1] $S_0(t)$ for the case without the heating pulse, and $S_1(t)$ for the case with the pulse. Because the heating pulse increases the current, and becuase the finite width of the pulse extends the duration of the plasma, a relative shift t' (=0.85 ms) of the time scales of these two curves is necessary before they can be compared. Fig. 1 (c) shows $S_0(t)$ and $S_1(t+t')$, and it may be seen that they are identical to within ± 2% in the range $2.6 \lesssim t \lesssim 4.8$ ms. The same time shift t' is also used in figs. 1 (a) and (b). For $t \lesssim 2.6$ ms transient phenomena such as skin current penetration into the plasma bulk (see below) are still incomplete; for $t \gtrsim 5$ ms (near the end of the discharge) the rapid and irregular decay of the discharge makes our measurements unreliable. But, we believe that in the range $2.6 \lesssim t \lesssim 4.8$ eq. (4) can be safely used in the determination of τ_E.

To obtain the electron temperature $\langle T_e \rangle$ we used Spitzer's resistivity formula [8], from which it follows that $\langle T_e \rangle \sim [I/(V - L\mathrm{d}I/\mathrm{d}t)]^{2/3}$. Neoclassical corrections of the formula are small for our discharge [8]. The results [‡1] with and without the heating pulse are shown in fig. 2. (The time shift t' described above has been incorporated in plotting the graph for the pulse-heated data.) The difference $\langle \Delta T_e \rangle$ between the two curves is proportional to ΔU_e. The slope of the curve of $\log\langle \Delta T_e \rangle$ versus t shown in the insert of fig. 2 gives an energy confinement time of $\tau_E = 0.9$ ms. This is to be compared to the value of $\tau_E \approx 0.7$ ms obtained from the con-

‡1 To compute S from eq. (3), the inductance L is needed. The value used, 0.15 μH, is the value for a parabolic profile, with adjustment for the geometry of our pickup loop. This choice is not critical since the $L\mathrm{d}I/\mathrm{d}t$ term is small compared to V in the time period of interest.

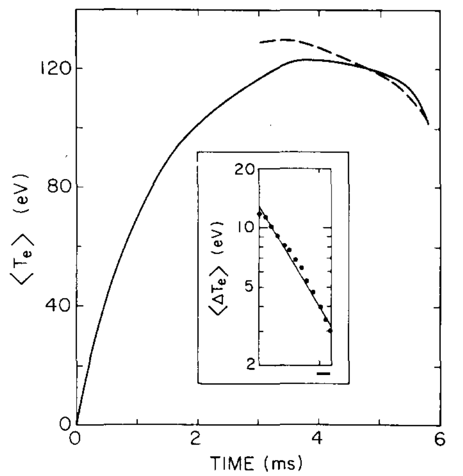

Fig. 2. Electron temperature $\langle T_e \rangle$ from resistivity, for the plasmas of fig. 1. In the insert the difference $\langle \Delta T_e \rangle$ between the two curves is plotted on a logarithmic scale (with the same time scale). The absolute value of $\langle T_e \rangle$ should be treated as an estimate that may be in error by as much as 20%; however, this does not affect the determination of τ_E.

ventional method based on the equation $\tau_E = U_e/VI$ applicable under steady-state conditions [‡2]. Such quasi-steady-state conditions exist in our own discharge at $t \approx 3.5$ ms.

The technique described here has three possible sources of error. First is the assumption that the heating pulse increases T_e without changing n_e (or other plasma parameters). This claim is supported by observations of the pulse's effect on the time evolution of ultraviolet lines [9] of O IV, O V, and O VI. Comparison of these measurements with computer solutions [9] of the rate equations for each species shows that, indeed, the major effect of the weak heating pulse is to alter the electron temperature.

Second is the phenomenon of skin penetration into the plasma bulk. In our discharge, calculations show that the skin time τ_s is ≈ 1 ms and one must therefore wait at least this long after the heating pulse before using eq. (4). After this period, the close agreement between S_0 and S_1 shown in fig. 1 (c) suggests that skin penetration is largely complete. Moreover, an analysis [10] of profile and skin effects shows that they contribute a small error ($<20\%$) to be observed τ_E even for waiting times less than τ_s. Since for tokamaks it is typical [1, 11] that $\tau_s \gtrsim \tau_E$ and τ_E can be tens of milliseconds, particularly for larger tokamaks, the waiting time can become unreasonably long. We point out, however, that the perturbing heating pulse can be generated in one of several other ways that do not require this long penetration time; for example, the plasma can be irradiated by a burst of microwaves.

Thirdly, we come to the determination of $\langle \Delta T_e \rangle$ from the electrical resistivity. No error in τ_E arises from an incorrect constant factor in $\langle T_e \rangle$ or $\langle \Delta T_e \rangle$. Some error does result, however, from differences between perturbed and unperturbed radial profiles of current density and electric field, and from variations of these perturbed profiles with time, especially due to skin penetration. Again our analysis [10] shows that errors in τ_E from these profile effects are small.

In conclusion, we have successfully tested a direct and relatively simple pulse-perturbation technique for determining the electron energy confinement time τ_E of a tokamak discharge. Improved accuracy from planned Thomson scattering measurements should give T_e to within 10%. The same method should yield $\langle \Delta T_e \rangle$ with a similar accuracy of 10–15%. This is the major source of error in a pulse perturbation measurement of τ_E. It is unlikely that similar accuracy can be achieved in the conventional technique, since this requires the determination of two parameters, n_e and T_e, plus their spatial profiles.

[‡2] To calculate U_e from eq. (2) we assumed parabolic profiles for T_e and n_e. Since these profiles have not been confirmed in our tokamak, the value of $\tau_E = 0.7$ ms must be treated as a crude estimate.

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