Anomalous impurity diffusion in models of tokamak edge plasma turbulence

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Using a simple spatially periodical and stationary potential as a model of electrostatic tokamak edge plasma turbulence, we have found recently [1, 2] an anomalous impurity diffusion in this regime. In this contribution, we estimate this diffusion for a more general and realistic form of the potential, which is close to the Hasegawa–Wakatani (HW) potential. As an interesting consequence of the discussed dynamics, we present possibility of radial electric field generation in the edge turbulence regime. This effect might play a role in tokamak scenarios with transport barriers.

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1 Introduction

It is generally accepted that tokamak edge plasma turbulence causes anomalous diffusion. Potential structures, formed as a result of plasma turbulence, are observed in the poloidal plane of tokamaks (e.g., [3, 4]), with typical poloidal correlation lengths
$\lambda = (10 \div 20)\,\mathrm{mm}$, lifetimes $\tau = (10 \div 20)\,\mu\mathrm{s}$, and amplitudes $U < 100\,\mathrm{V}$. Theoretical studies, addressing the anomalous diffusion in these fields are usually based on the test–particle drift approximation and on the electrostatic field resulting from the Hasegawa–Mima model (see, e.g. [5]) or the Hasegawa–Wakatani model [6].

In our preceding papers [7, 8], we discussed the effect of the anomalous ion diffusion on radial electric field generation. There we used a very simplified model of the turbulent potential structures, namely, a spatially periodic and time–independent

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potential. Using a Hamiltonian approach (which also takes into account the cyclotron motion), we have found, for impurity ions C^+ and usual potential amplitudes, a substantial increase in the diffusion of these ions (both of the Gaussian and Lévy–walk forms), resulting in the radial electric field. Using a drift approximation for this case, no diffusion and no electric field is observed. Hence, there is a considerable difference between these two approaches, which implies an uncertainty in using the drift approximation. Therefore, we decided to use the Hamiltonian approach also in the more realistic models of the edge plasma turbulence; the results will be then compared with results, following from the drift approximation approach.

2 Diffusion in time independent and spatially periodical potential and in homogeneous magnetic field

To understand more properly to the effect of anomalous diffusion of ions, found in the regime of edge tokamak plasma turbulence, we have recently [1, 2] started with very simple model of the turbulent potential. Instead of obviously time and spatially uncorrelated dependencies, we use the spatially periodical model shown in Figs. 1a, 1b. The first figure presents the axonometric view of the potential in the poloidal landscape, the second its system of equipotentials and separatrices. Since we considered the time–independent case, the drift trajectories must follow the equipotentials.

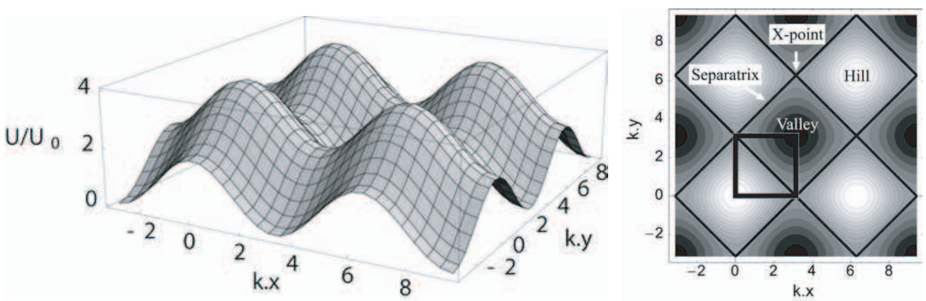

Fig. 1. Axonometric view of the spatially periodical potential and its system of equipotentials and separatrices.

Nevertheless, the Hamiltonian description of the motion brings another picture. To show that, we discuss the following Hamiltonian

$$H = \frac{1}{2m}(p_x - qA_x)^2 + \frac{1}{2m}(p_y - qA_y)^2 + q\phi(x, y, t)\,.$$

Here, x, y are 2D spatial Cartesian coordinates, p_x, p_y are their canonically conjugated momenta and m is the mass and q is the charge of the ion. Expressing the components of the vector potential $\mathbf{A}$ by means of the magnetic field $\mathbf{B}$, we obtain

$$H = \frac{1}{2m}(p_x + \frac{m\omega_c}{2}y)^2 + \frac{1}{2m}(p_y - \frac{m\omega_c}{2}x)^2 + q\phi(x, y, t)\,.$$

Here, ω_c, the cyclotron frequency, is defined as $\omega_c = \frac{qB}{m}$, where B is the intensity of the homogeneous magnetic field. In our model, we assume the potential in the form

$$\phi = \phi_0(2 + \cos kx + \cos ky)\,.$$

The particle dynamics is then fully determined by their initial conditions and by the dimensionless parameter $R = \frac{Am_pU_0k^2}{Zq_pB^2}$, where A, Z being the mass and charge numbers, respectively, ϕ_0 is the amplitude of the potential and k is the wave number of the periodicity.

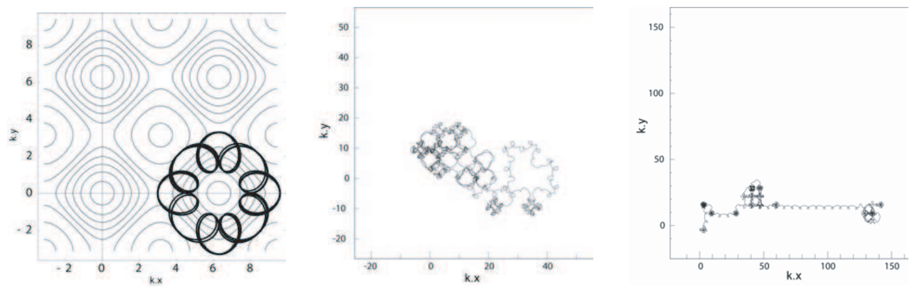

Fig. 2. Three types of motion of ions in the spatially periodical and time–independent potential and in an homogeneous magnetic field: **a)** regular motion along the equipotentials. Diffusion of a Gaussian type. **b)** particles move along separatrices and in the x–points jump in the neighbouring cell. **c)** intermittent trapping and ballistic motion of particles. Diffusion of the Lévy walk type.

Depending, e.g., on amplitudes of the potentials, we obtained three different types of motion. For small amplitudes (Fig. 2a), the full motion consists of cyclotron circles, drifting along the equipotentials. Only this case fully corresponds to the drift description. For larger amplitudes, particles start to diffuse along the potential landscape (Fig. 2b), jumping in the x regions chaotically in the neighbouring cells. This motion can be considered as gaussian diffusion. For even larger amplitudes, the motion starts to have strange dynamics character. (According, e.g., [8, 9] the strange dynamics is characterized by variance with time dependence $\approx t^\gamma$ with $1 < \gamma < 2$. The case $\gamma = 2$ corresponds to the ballistic motion, the case $\gamma = 1$ to the Gaussian diffusion). A typical example of this motion is presented in Fig. 2c. The dynamics of particles in this simple systems is quite complicated. There are fundamental differences between Hamiltonian and drift description. Putting a particle on the hill or in the valley and using the drift description, in both cases the particle will circulate at the original potential level with no hope for jumping into the neighbour cell. On the other hand, using the Hamiltonian approach, the dynamics can be changed considerably. Considering a particle travelling on the hill, there appear, namely, two regimes of its motion. According to paper of Bellan [11] and of our paper [1], there appears an instability for the case with $R \geq 0.25$. A particle with this parameter spirals downhill until it enters the separatrix region. (For $R \leq 0.25$ the particle remain to move at the same potential level). Once in the separatrix region, the particle starts chaotically move among neighbouring cells, diverting just in the region of x–point. On the contrary, a particle in the valley has no chance to jump out and will be definitely trapped. Since R depends linearly on the mass, motion of ions and electrons quite differs. Supposing that ions satisfy the condition $R \approx 0.25$ (and are, therefore, in a chaotic regime), electrons are deeply in the regular drift motion. The diffusion of ions is, therefore, not accompanied by the diffusion of electrons and a deficit of the positive charge appears. This led us to the idea of a generation of radial electric field in this model.

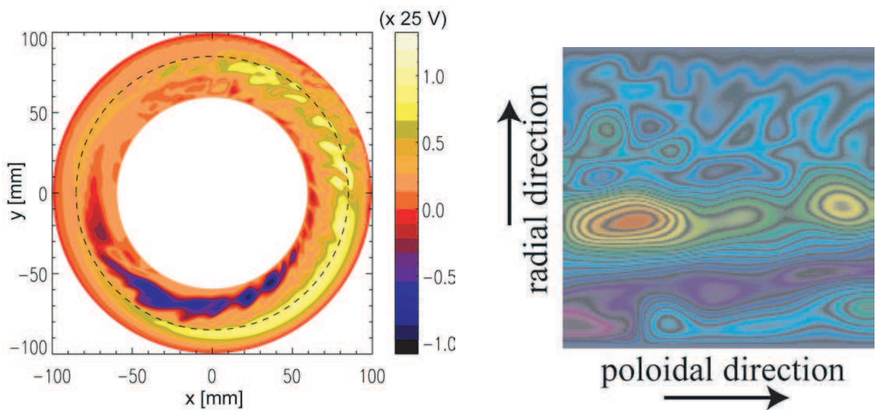

Fig. 3. Potential relief for the Hasegawa–Wakatani model for CASTOR tokamak parameters.

3 Anomalous diffusion of impurities and generation of radial electric field in numerically simulated turbulent potential

In the work described above, the anomalous diffusion was studied in a rather simple approximation of the turbulent potential, appearing in the edge plasma. In this section, a more realistic form of this potential will be considered. For the model, we take an approximated form of Hasegawa–Wakatani equations and corresponding potential, described in [12] and use the parameters set for CASTOR tokamak. In what follows, we use this potential only as a test–bed for the discussion of particle dynamics. Fig. 3 shows the form of the potential, Fig. 4 the typical potential structure with hills and valleys, reminding the periodical structure, discussed above. In this potential we follow particles (C^+ ions and electrons), using both the full Hamiltonian solution and the drift approximation. For electrons, due to their low mass, both approximations can be approximately considered as the same. Fig. 5 shows and example of C^+ ions trajectories for both the Hamiltonian solution and the drift approximation. There are two set of trajectories for two different initial conditions. The diversity of both solutions is remarkable and confirms our opinion concerning necessity of using the Hamiltonian approach. Using, as usual, the corresponding variances for determination of diffusion coefficients, we found that the diffusion coefficient for C^+ ions is approximately twice larger than that one for electrons. Consequently, the conditions for the generation of the radial electric field are fulfilled. Since the generation of the electric field due to different diffusion of ions and electrons is quite complex problem requiring the self–consistent solution, we can only very roughly estimate the amplitudes at this time. In our case, we expect the amplitude of the order 1 kV/m. However, the self–consistent model is now in preparation.

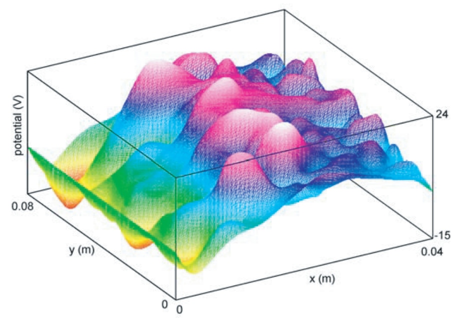

Fig. 4. Nonlinear potential structure.

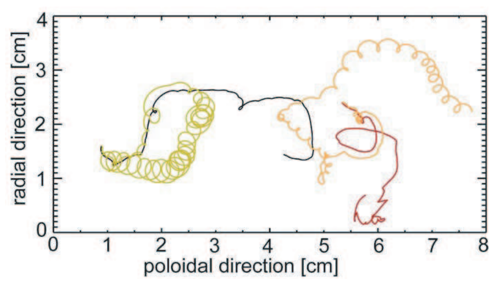

Fig. 5. Trajectories for the Hamiltonian and drift approximations.

For better illustration of the different character of the particle dynamics in periodical potential and more realistic time–dependent potential we show Figs. 6 and 7 where are presented time–shots of particle diffusion in these potentials. Fig. 6 represents diffusion of particles in the periodical potential and Fig. 7 the same for the turbulent potential.

4 Conclusions

The main result of the paper is finding that in the turbulent plasma, the generation of the electric field can be caused due to the different diffusion of ions and electrons, appearing only in the Hamiltonian approach. The first quantitative estimation provides radial electric fields of the order of 1 kV/m. Quantitatively, the confirmation of this idea requires recalculation of our model for potentials, obtained from more exact forms of numerically solved turbulent potentials and using self–consistent approach. This is under study now.

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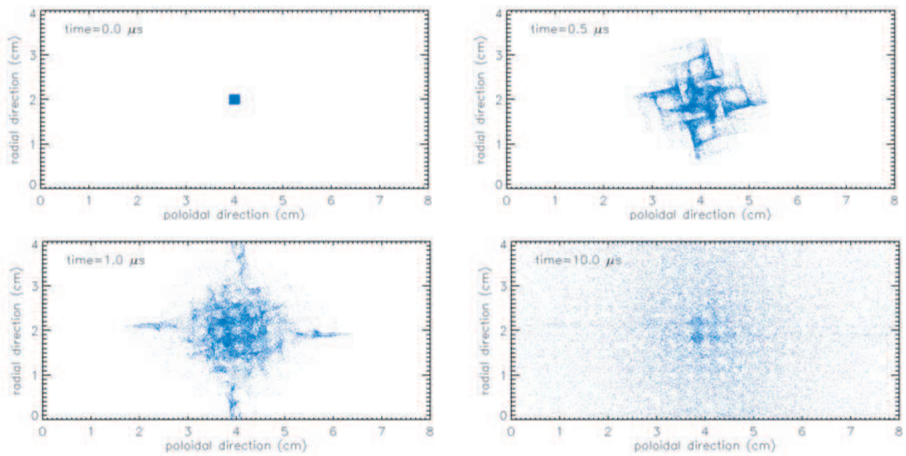

Fig. 6. Time sequences for the diffusion of carbon impurities in the spatially periodical and stationary potential.

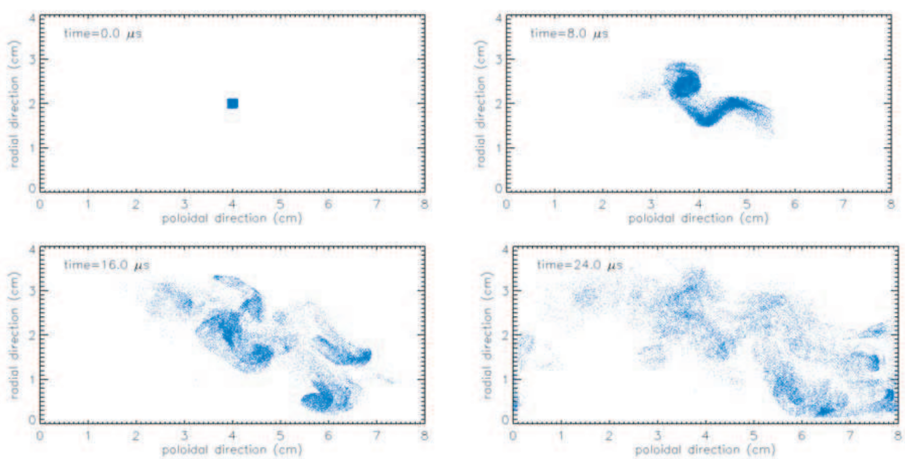

Fig. 7. Time sequences for the diffusion of carbon in Hasegawa–Wakatani potential; an analogy to the same sequence in spatially periodical and stationary potential.

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