We have also tried to observe the emission bands of molecular hydrogen, but they were too weak to be detected.

In this Letter, we have demonstrated the feasibility of a method of determining n_H, n_{H2} and n_e from the observed intensities of atomic emission lines. An improved data set of the relevant cross-sections would further improve the accuracy of the present technique.

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NUMERICAL TRANSPORT STUDIES FOR THE CASTOR TOKAMAK

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ABSTRACT. A transport code simulation of the plasma evolution in the CASTOR tokamak has been performed, which is in agreement with the available experimental data. A method of determining the empirical model for the electron thermal diffusivity is presented. The transport coefficient is consistent with the expectations of the electrostatic drift wave theory.

The computer simulation of transport processes in a tokamak plasma is closely related to theoretical and experimental studies. One problem is the computation of measurable plasma parameters from a given physical model for the transport coefficients. The inverse problem arises when experimental data from a tokamak discharge are used to determine the empirical values of transport coefficients. Because of the complexity of the tokamak phenomena, theoretical models are usually unable to describe plasma dynamics adequately so that it is very useful to obtain empirical models by solving the inverse problem. To determine the appropriate transport coefficients, transport codes are necessary tools in experimental tokamak studies.

The results of computer simulations of the inverse problem are presented for the CASTOR tokamak (a small device with $R = 0.4$ m, $a = 0.075$ m and $B = 1.3$ T) [1, 2]. They are obtained in two stages, a method which makes this very difficult problem more tractable. The first stage is based on a chain of interconnected, simplified models and the corresponding small computer codes. A large number of iterative trials can easily be performed to determine the empirical transport model as well as some plasma parameters which are, in the context of this model and with our simplifications, consistent with the experimental data. With this preliminary information, in the second stage an extended computer code based on a self-consistent system of transport equations can be used to improve and complete the results of the first stage.

An important observation is that this method works even with a small amount of experimental data. This is precisely due to the use of the interconnected partial models which clearly reveal the detailed correlations between the plasma parameters (which are, to a large extent, hidden in the complex, self-consistent transport description).

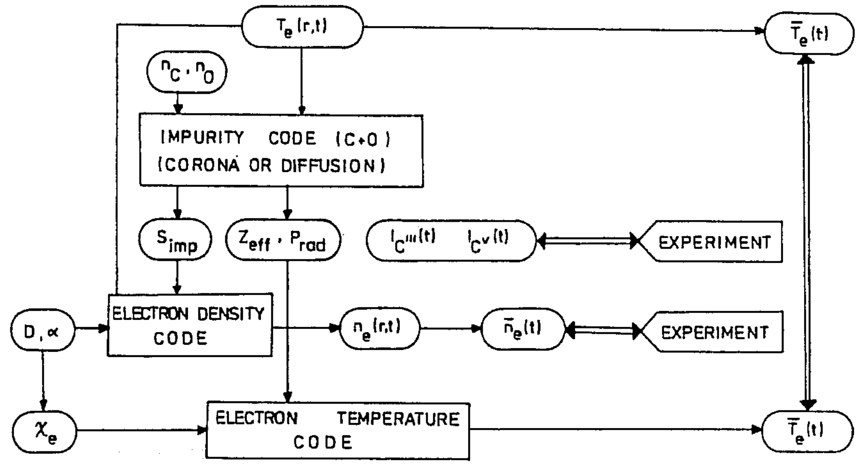

FIG. 1. Interconnection scheme for simplified calculations.

The particular interconnection scheme for the simplified calculations is presented in Fig. 1. It is based on the experimental data available for the numerical simulation of the transport processes in the CASTOR tokamak, i.e. the temporal evolution of the total plasma current and the electron density (which, after a fast growth, instead of the usual quasi-stationary regime, shows an exponential decay — see Fig. 2) and the intensities of two carbon impurity lines (C III and C V) integrated over the plasma diameter (Fig. 3). The main modules of the scheme concern the evolution of the

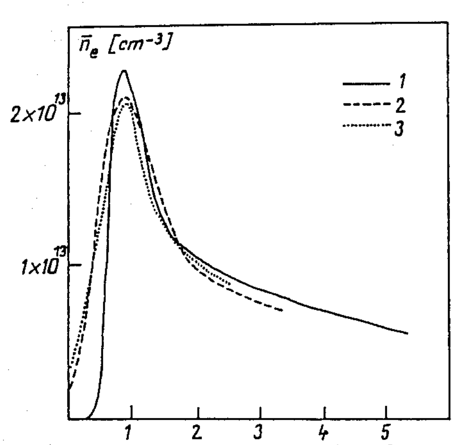

FIG. 2. Time evolution of the average electron density in the 12 kA discharge: (1) experimental; (2) result of the simplified model; (3) result of the full transport code.

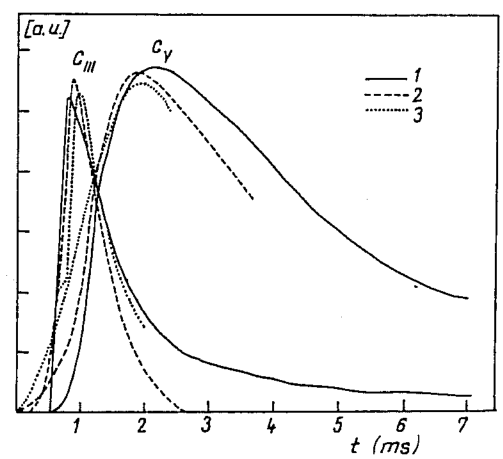

FIG. 3. Line radiation intensities from C III (λ = 2269 Å) and C V (λ = 2270 Å): (1) experimental; (2) result of the simplified model; (3) result of the full transport code.

impurity and electron densities. The most important unknown parameters are the electron temperature, $T_e(r, t)$, and the particle diffusion coefficient, $D(r, t)$. The functions $T_e(\vec{r}, t)$, which allow a satisfactory agreement to be obtained between computed and measured data for the integrated emissivities of the C III and C V lines, enter the second module, both directly and through the electron source from impurity atom ionization, S_{imp}. The particle diffusion coefficients, $D(r, t)$, which lead to a good fit of the experimental average electron density, are determined in the electron density module. To approximately satisfy the self-consistency of the results of these calculations, a third module is introduced. It solves the zero-dimensional energy balance equation for the electrons, using the effective charge and the radiation losses as determined in the first module and a thermal diffusivity proportional to the electron particle diffusion coefficient obtained in the second module. Self-consistency is fulfilled when the average electron temperature, $\bar{T}_e(t)$, as determined by this module, is in reasonable agreement with the average of the input function, $T_e(r, t)$.

A very long series of iterative computations was performed with these interconnected modules.

For most calculations, the impurity module contained the simplest description of the line radiation from the carbon and oxygen atoms, i.e. the coronal equilibrium model. This simplification is justified by the computations we have performed with more elaborated impurity codes [3]. For each charge state, they take into account, in addition to ionization and recombination processes, the diffusion as given by the usual neoclassical model. These studies show that diffusion has an important effect on the spatial distribution of impurity densities, but that it only results in a small temporal shift of the line integrated intensity maxima.

In the electron density module, the equation of diffusion for the electrons,

$$\frac{\partial n_e}{\partial t} + \frac{1}{r}\frac{\partial}{\partial r}(r\,\Phi) = n_e n_0 \langle \sigma_{ion} v(T_e)\rangle + S_{imp} \qquad (1)$$

is solved. Here, Φ, the electron particle flux, is of the form

$$\Phi = -D\frac{\partial n_e}{\partial r} + n_e\tilde{\Gamma}, \quad \tilde{\Gamma} = \alpha\frac{E}{B_\theta}\left(\frac{r}{R}\right)^2 \qquad (2)$$

In the different runs, the particle diffusion coefficient $D(r, t)$ is introduced by an analytical expression that depends on a few numerical parameters:

$$D(r, t) = D_{PS}(r, t)\,(1 + c(1 - \exp(-t^2/\beta))) \qquad (3)$$

where D_{PS} is the Pfirsch-Schlüter diffusion coefficient. The anomalous inward pinch $n_e\tilde{\Gamma}$ [4] is calculated with the toroidal electric field, $E = \eta J$, and the poloidal magnetic field, $B_\theta = (\mu_0/r)\int_0^r J(r')\,r'\,dr'$, obtained from a current density $J(r, t)$ proportional to $T_e^{3/2}$ and subject to the constraint $\int_0^a J\,2\pi r\,dr = I(t)$, where $I(t)$ is the plasma current. η is the Spitzer resistivity with a neoclassical correction. Since the CASTOR tokamak plasma is collisional, low values were attributed to the parameter $\alpha (0 < \alpha < 10)$; $\langle\sigma_{ion} v(T_e)\rangle$ is the ionization rate, and $n_0(r, t)$ is the neutral atom density modelled by the expression:

$$n_0(r, t) = N_0 \exp(-t/\tau_2)\exp(-(a - r)/r_0) \qquad (4)$$

The values of N_0, τ_2 and r_0 appropriate for the description of the CASTOR discharge were found to be in the ranges $(0.6\text{–}1) \times 10^{13}\,cm^{-3}$, 0.4–0.6 ms and (0.2–0.3) a, respectively.

The most important result of the study performed with the interconnected models is that the experimental data can be fitted simultaneously with only one pair of functions, $T_e(r, t)$ and $D(r, t)$; these satisfy the self-consistency condition approximately. Since the experimental data for each module are profile averages, they are compatible with several fitting functions, but the whole connected scheme eliminates all but one solution. This solution corresponds to the plasma being heated non-uniformly over the cross-section, with a warm central channel developing from the beginning of the discharge (reaching about 250 eV in 2 ms). A low temperature outer region, extending over approximately half the plasma radius, determines the main contribution to the line radiation from impurities. High particle losses occur; they are represented by a large value obtained for the parameter c in Eq. (3) ($c \cong 1000$ and $\beta \cong 8$, with t in ms).

The second stage in the transport studies relies on the set of partial differential equations [5], representing the diffusion of electrons (1), the energy balance for electrons (5) and ions (6), and Maxwell's equations, (7) and (8):

$$\frac{3}{2}\frac{\partial}{\partial t}(n_e T_e) = -\frac{1}{r}\frac{\partial}{\partial r}\left(r\left(q_e + \frac{3}{2}n_e V T_e\right)\right) - 3\frac{m_e}{m_p}\frac{T_e - T_i}{\tau_e}[Z]n_e + EJ - Q_{rad} - f(t)n_e n_0\langle\sigma_{ion} v(T_e)\rangle \qquad (5)$$

$$\frac{3}{2}\frac{\partial}{\partial t}(n_e T_i) = -\frac{1}{r}\frac{\partial}{\partial r}\left(r\left(q_i + \frac{3}{2}n_e V T_i\right)\right) + 3\frac{m_e}{m_p}\frac{T_e - T_i}{\tau_e}[Z]n_e - n_p n_0 R_{cx}\frac{3}{2}(T_i - T_0) + n_e n_0\langle\sigma_{ion} v(T_e)\rangle\frac{3}{2}T_0 \qquad (6)$$

$$J = \frac{1}{\mu_0}\frac{1}{r}\frac{\partial}{\partial r}(rB_\theta) \quad (7)$$

$$\frac{\partial B_\theta}{\partial t} = \frac{\partial E}{\partial r}; \qquad E = \eta J \quad (8)$$

Here, τ_e is the inverse of the electron–ion collision frequency ν_e, R_{cx} is the charge exchange rate and V is the radial diffusion velocity obtained from $n_e V = \Phi$. The thermal conductivity fluxes have the form $q_a = -n_e \chi_a \partial T_a/\partial r$, with a = e, i, and χ_a being the corresponding diffusivity coefficient. The coefficient $f(t) = 13.6\ (1 + 15\ \exp(-t/\tau_1))$, with $0.2\ \text{ms} \leq \tau_1 \leq 0.3\ \text{ms}$, is introduced in the power lost by electrons through neutral ionization in order to account for the complexity of atomic processes in the initial stage which determine the large energy losses for each new electron–ion pair. T_0 is the temperature of the neutral atoms; it is usually taken to be a fraction of the ion temperature, $T_0 = T_i/10$. Equation (6) represents the energy balance for all ion species in the plasma. It is obtained by summing up the energy conservation equations written for each ion species (including protons) multiplied by the ion charge. The following assumptions are made: charge neutrality, flux ambipolarity and thermal equilibrium between the ion species. The same diffusivity for all ions is also assumed in Eq. (6), which is an acceptable approximation because of the small contribution of the impurities to the thermal energy transport. χ_i is neoclassical [6], and n_p is the proton density obtained from the condition of charge neutrality. The effective charge weighted by the inverse of the atomic mass [Z] [7] and the power lost by radiation, Q_{rad}, are determined from the impurity module of the code. It solves a diffusion equation for each impurity species and obtains the densities for the charge states on the assumption of corona equilibrium at each point inside the plasma.

The simulations performed on the basis of a theoretical model appropriate to the CASTOR tokamak (neoclassical collisional transport coefficients [6]) cannot fit the experimental data (e.g. for average density). Moreover, the central region is heated rapidly; with time, this leads to unphysical, very narrow, highly peaked electron temperature and current density profiles. Neither a runaway component in J nor radiation from impurities can prevent the explosive evolution of the central channel. Another possibility is to investigate the influence of some plasma instability in this low temperature, high collisionality regime; possible instabilities are of the dissipative, electrostatic type [8]. It has been demonstrated [9] that a high electron temperature gradient destabilizes the dissipative drift wave. When the confining magnetic field has low shear, the unstable mode has a growth rate

$$\gamma \cong (k_\perp \rho_s c_s/L_T)^2/\nu_e \quad (9)$$

where $k_\perp$ is the wave number normal to the confining magnetic field, L_T is the characteristic length of the electron temperature variations, c_s is the ion sound speed and $\rho_s = m_e c_s/eB$ is the ion Larmor radius based on T_e. The thermal diffusivity is estimated to be $\chi_e = 5D$, from Kadomtsev's formula ($D = \gamma/k_\perp^2$). Within this model, the simulation clearly shows that no explosive evolution of the central channel occurs and energy is transferred to the cold plasma of the outer region. However, the experimentally established decay of $\bar{n}_e$ cannot be obtained with this transport model, which suggests that it is, in this case, very difficult to obtain results close to the experimental ones by numerical simulations based on theoretical models.

As a result, the inverse problem must be solved and an empirical model for the transport coefficients (i.e. for D, with χ_e being related to D and χ_i remaining neoclassical) must be determined. The estimate for D(r, t) as obtained at the previous stage (Eq. (3)) is essential since the trials for fitting the experimental data use functions that are close to this estimate. Indeed, a good fitting of the experimental data was obtained with the full transport code, confirming that the simplified calculations not only constitute a useful stage in solving the problem but also can provide a reasonable estimate for the plasma dynamics.

The electron thermal diffusivity determined by the self-consistent transport calculations is presented in Fig. 4. The results obtained for the plasma parameters are shown in Figs 5–7. They show the development of a warm central channel, the peaking ratio for the electron temperature and for the current density saturating at about three to four (higher than two, which corresponds to parabolic profiles). With the help of this model the simulation is consistent with other experimental measurements, such as that of the loop voltage, total radiated power, electron density and temperature at the plasma edge.

It is conceivable that these calculations represent the starting point for elaborating a theoretical model for the CASTOR discharge. We conjecture that electrostatic drift wave turbulence develops gradually in time, reaching a maximum level at about 1.5 ms. Rough estimates based on the 'mixing length' assumption [8] determine the level of density fluctuations to be

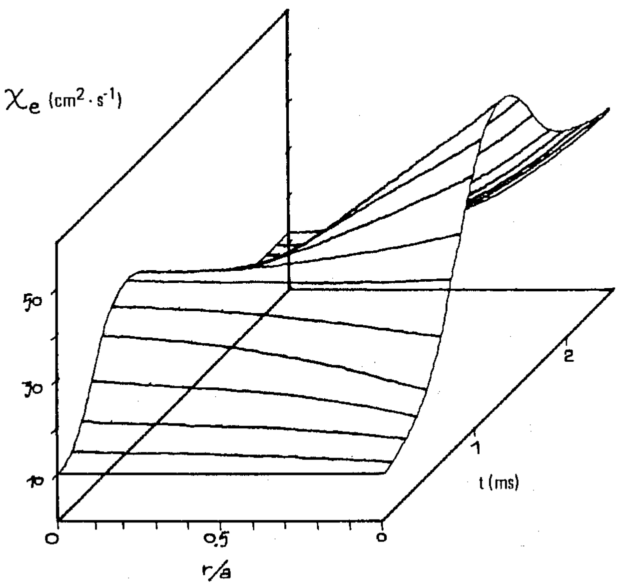

FIG. 4. Thermal diffusivity for electrons obtained for the CASTOR discharge.

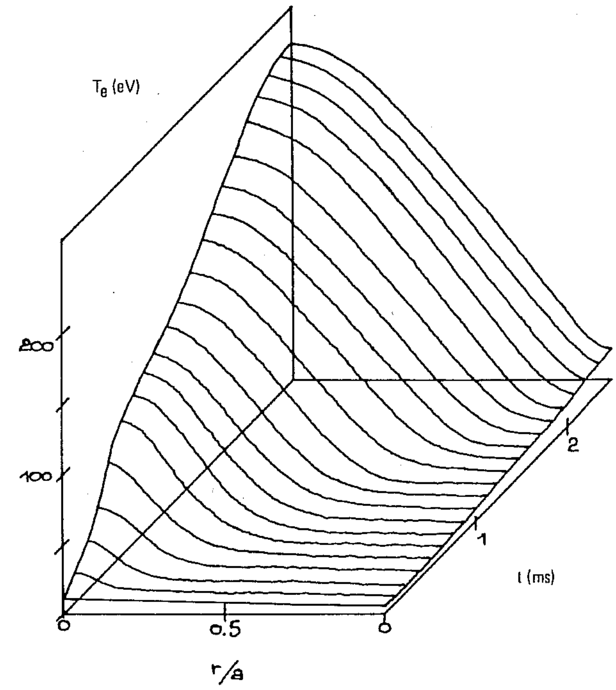

FIG. 6. Electron density determined for the CASTOR discharge.

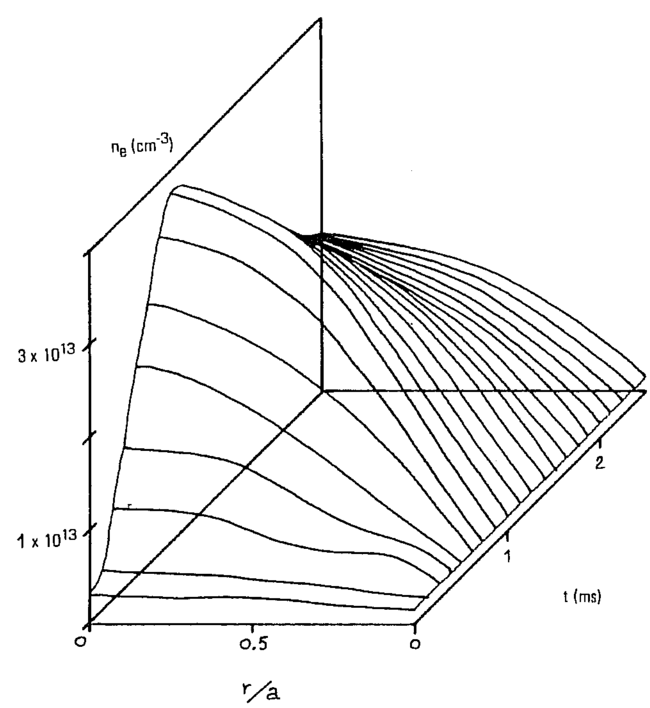

FIG. 5. Electron temperature determined for the CASTOR discharge.

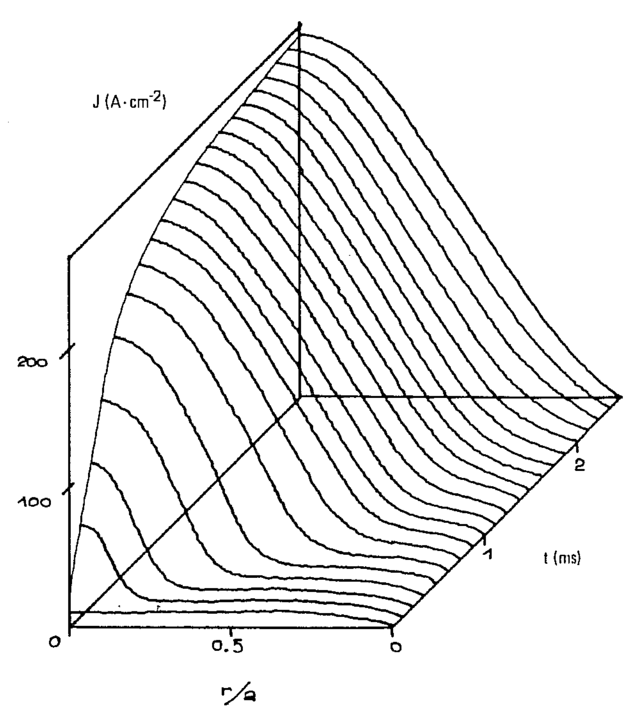

FIG. 7. Current density determined for the CASTOR discharge.

$\tilde{n}_e/n_e$ = 80–100% in the outer region of the plasma and $\tilde{n}_e/n_e$ = 10–20% in the central channel. These values, obtained from the calculated anomalous electron diffusivity, are in accordance with recent experimental measurements of density fluctuations in the CASTOR tokamak [10].

In conclusion, we can say that we have made predictions for certain unmeasured plasma parameters (such as the electron temperature) on the basis of a realistic transport model of the CASTOR discharge which is consistent with the expectations of electrostatic drift wave theory. This procedure of numerical transport studies could also be used for other devices and has the advantage of providing a better understanding of the parameter correlations during discharge evolution.

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