Momentum and Heat Transfer From Lower Hybrid Antennas to the Tokamak Edge Plasma*

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1 INTRODUCTION

Localised heat loads on tokamak first wall and/or divertor components has been observed on a number of tokamaks during operation with Lower Hybrid Current Drive (LHCD) [1-3]. Evidence from Langmuir probe [2] and direct infrared imaging of locations magnetically connected to the LH grill mouth [3] supports the suggestion [1,2] that the damage is at least partially caused by fast electrons generated in front of the LH antenna. Observations of stationary hot spots on the guard antenna limiter during LHCD with a steady LH power input, combined with floating potential measurements at the ergodic divertor plate, indicate a steady fast electron flow for the duration of the LH pulse [4].

A theory to explain the origin of the fast electrons has been previously developed on the basis of a Landau [5] and/or Fermi [6] interaction of thermal edge electrons with the LH antenna electric field. Momentum and energy is transferred from the antenna field to resonant edge electrons through the Landau interaction. Energetic electrons streaming outward from either end of the grill are thus generated. For typical grill electric field intensities around 3-5 kV/cm the higher harmonics of both the forward and backward spectra experience overlap (in the Chirikov [7] sense) in velocity space.[5] The globally stochastic region extends from the electron Landau damping limit $v_{//}/v_e \cong 3$ up to the overlap limit which for the forward spectrum is about 2 keV for an electric field of 4 kV/cm [5].

Here we examine in more detail the momentum and energy transfer from the antenna field to the edge electrons. We derive the LH-induced electron diffusion coefficient, and then find the exact fast electron distribution function from the collision-less quasi-linear kinetic equation with parallel flow. Ensemble averages $\langle v_{//}\rangle$ and $\langle v_{//}^2\rangle$ are compared with simulation results. The LH force acting on electrons, the dissipated LH power, the fast electron power flow in front of the antenna, and the radial width of the interaction region are calculated analytically.

2 FAST ELECTRON DIFFUSION COEFFICIENT

The quasi-linear diffusion coefficient $D(v_{//})$ in front of the LH grill is obtained by integrating the equation of motion

$$\ddot{z} = \dot{v} = \frac{e E_0}{m_e} A(z) \cos \Phi(z,t) \equiv \omega v_q A(z) \cos\left[\omega t - \varphi(z) + \varphi_r\right] \qquad (1)$$

along unperturbed electron trajectories $z = v_{//} t$, and then carrying out a simple resonance broadening correction. The diffusion in an ensemble of particles is generally defined as $D(v_{//}) = \langle \Delta v_{//}^2 \rangle / 2t$ where <…> signifies here ensemble averaging with respect to a random initial phase φ_r which distinguishes the individual electrons. Further, $\omega = 2\pi f$ ($f = 3.7$ GHz), and $v_q = eE_0/m_e$ is the electron quiver velocity. For the purpose of calculating D we consider the simple case of a grill fundamental mode i.e. A(z) = 1 in front of the wave-guide mouths and A(z)=0 in front of the septa. Further, we take $\pi/2$ phasing between the 32 wave-guides of the Tore Supra antenna [6].Thus from (1) the change in electron velocity across any of the wave-guides is

$$\Delta v_{//} = 2\, v_q \sin\Omega\, \cos[(2j-1)\Omega+\varphi_r], \quad \Omega = \frac{\omega d}{2 v_{//}} - \frac{\pi}{4}, \quad j = 1,2,\ldots,32 \qquad (2)$$

which upon squaring, summation and ensemble averaging over the random phases φ_r yields :

$\langle\Delta v_{//}^2\rangle = 2\, v_q^2 \sin^2\Omega$. Since the electron transit time across a wave-guide is $\tau_c = d/v_{//}$, the diffusion coefficient is $D(v_{//}) = (v_q^2/d)\, v_{//} \sin^2\Omega$.

For diffusion to occur, the antenna field intensity must satisfy the resonance overlap condition $4\omega d v_q / v_{//}^2 \geq 1$ (i.e. mode separatrix width about equal to the distance between neighbouring modes). In the globally stochastic region, which clearly is bounded, we can average the quasi-linear D over the resonant peaks and set D to zero outside the stochastic region. Since D must be non-negative, this finally gives D_{ql} and a diffusion coefficient $D(v_{//},t)$ in the following form :

$$D_{ql} = v_q^2\, |v_{//}|/2d \equiv |v_{//}|\, D_0 \quad \text{and} \quad D(v_{//},t) = s(t)\, D_{ql}; \qquad v_{ELD} < |v_{//}| < v_{max}, \qquad (3)$$

where $v_{ELD} \cong 3\, v_e$ is the electron Landau damping limit, v_{max} is the resonance overlap limit depending on the grill electric field intensity [5], and s(t) is some decreasing function of time resulting from a velocity variance which in a bounded velocity space region must saturate in time. Figure 1 shows the ratio D_{sim}/D_{ql} with D_{sim} obtained from test electron simulations of an ensemble whose dynamics is described by Eq. (1). We do not yet have a theory to describe the decay in time of the field autocorrelation function in a bounded velocity region so that we approximate s(z) by a profile taken from D_{sim} and demand that initially s = 1.

3 THE FAST ELECTRON DISTRIBUTION IN FRONT OF THE GRILL

The fast electron distribution function f satisfies the quasi-linear Fokker-Planck equation

$$\mathrm{div}_v\, \vec{S}_{coll} + \mathrm{div}_v\, \vec{S}_{rf} + \mathrm{div}_r(\vec{v}f) + \mathrm{div}_v(\tfrac{e}{m}E_{sep}f) = 0 \qquad (4)$$

where $S_{rf} = -\partial_{//}(D_{LH}\partial_{//}f)$, D_{LH} is the (zz) element of the diffusion tensor, and E_{sep} is a charge separation field [8] which we here disregard. The effect of collisions can be neglected since the fast electron propagation time along a magnetic field line connecting the grill to a divertor target (about 1m in Tore-Supra) is about $\tau_{conn} \cong 10^{-8}$s, whereas the slowest collision time τ_{coll} experienced by the fast electrons, i.e. the fast electron – thermal electron collision time, is $\tau_{coll} \cong 7\times10^{-5}$s . We first deal with $f_\perp$, assuming separability of the distribution function in velocity space, i.e. $f = f_{//}f_\perp$. The fast electron perpendicular dynamics is effected neither by collisions nor by the antenna electric field so that $f_\perp$ retains a thermal Maxwellian character with $T_\perp = T_e$ ($\cong$ 25 eV). The parallel $f_{//}$ is likewise unaffected by collisions but is strongly modified by the RF-driven diffusion within the stochastic region. The fast electron distributions $f_{//}^{(+,-)}$ are one-sided, i.e. are defined on parallel velocity half-space. They are fed by point sources of thermal electrons $\delta(v_{//} - v_{//0})$, situated at either end of the grill $z = z_0$ or z_L. The distribution functions $f_{//}^{(+,-)}$ can therefore be represented by Greens' functions of Eq. (4) where we use the quasi-linear result (3) : $D = D_0\, s(z) v_{//}$ with $D_0 = v_q^2/2d_G$ and s(z) a suitable decreasing function in the spectrum direction. We next drop the perpendicular part of the spatial flow divergence term. Finally, the transformation $\zeta = F(z)-F(z_0)$, where F(z) is the primitive function to s(z), will yield the following equation for the Green's function G

$$\frac{\partial^2 G}{\partial^2 v_{//}} + \frac{1}{v_{//}}\frac{\partial G}{\partial v_{//}} - \frac{1}{D_0}\frac{\partial G}{\partial \zeta} = -\frac{\delta\{F^{-1}[\zeta + F(z_0)] - z_0\}\delta(v_{//} - v_{//0})}{D_0\, v_{//o}} \qquad (5)$$

Equation (5) is defined on $\zeta \in <0,\infty)$ so that it can be Laplace – transformed [9] in ζ, leading to a Bessel equation for the image Green's function

$$\frac{\partial^2 \tilde{G}}{\partial^2 v_{//}} + \frac{1}{v_{//}}\frac{\partial \tilde{G}}{\partial v_{//}} - \frac{p}{D_0}\tilde{G} = -\frac{p\,\delta(v_{//} - v_{//0})}{D_0 v_{//o}} \qquad (6)$$

The solution of Eq.(6) which vanishes as $v_{//} \to \mp\infty$ is the modified Bessel function $(p/D_0)K_0[|v_{//}|(p/D_0)^{1/2}]$ which on inverting [9] gives the half-space solution

$$G(v_{//}, z) = \frac{1}{D_0\zeta}\exp\left[-\frac{v_{//}^2}{4D_0\zeta}\right] \quad ; \quad \text{and} \quad G = 0 \quad \text{on the other side of } v_{//} = 0 \qquad (7)$$

The fast electrons therefore have a Maxwellian distribution f with a parallel temperature $T_{//}(z) = 2D_0\zeta$ and we take $f^{(+,-)}=n^{(+,-)}G^{(+,-)}$ where $n^{(+,-)}$ represent, respectively, the hot forward and backward population densities. Velocity space averages are defined in the usual manner as

$$< A > = \int G A\, dv_{//} \Big/ \int G\, dv_{//} \qquad (8)$$

In the following we make the specific choice $s(z) = z_0/z$ which gives $T_{//} = 2D_0 z_0 \ln(z/z_0)$. For the grill entry point we obtain z_0=0.11m from Fig.1. This fixes our reference system which so far was arbitrary. We obtain the averages $<|v_{//}|> = \sqrt{(2T_{//}/\pi)}$ and $<v_{//}^2> = T_{//}$. The theory and simulation results for $<v_{//}^2>$ are compared in Fig. 2.

4 MOMENTUM AND ENERGY BALANCE EQUATIONS

The fast electron parallel force and power balance equations are respectively the $mv_{//}$ and $mv^2/2$ moments of the kinetic equation (4). Using the distribution function (7) with (8) gives

$$F_{rf}^{(+,-)} = m D_0 (ns)^{(+,-)} \operatorname{sign}(v_{//}) = \partial_z (nW)^{(+,-)} \qquad (9a)$$

$$p_{rf}^{(+,-)} = 2mD_0 (ns)^{(+,-)} < u > = \partial_z (nS)^{(+,-)} \qquad (9b)$$

where $W = (1/2)\, mT_{//}$, $S_{//}(z) = (2/3)\, m<u>T_{//}$, $u = |v_{//}|$, F_{rf} is the RF-induced force density, and p_{rf} is the dissipated RF power density. In Eq. (9b) we have neglected terms depending on perpendicular temperature on account of $T_\perp=T_e << T_{//}$ .We recall that the functions s(z) decrease, and $T_{//}(z)$ and the parallel flow density $S_{//}(z)$ increase, in the direction of the resonant electron forced motion. The mirror image of $s^{(+)}(z)$ with respect to the grill center is $s^{(-)}(z) = 1-z_0/z+z_0/z_L$, where $z_L=z_0$+(grill length). The equalities (9) indicate how the induced kinetic terms depend on the electron diffusion and essentially express the effect of the resonant interaction in fluid terms.

The total electron force and power balance equations are

$$F_{rf}^{(+)} - F_{rf}^{(-)} = \partial_z [n(W^{(+)} + W^{(-)})] \equiv \partial_z (nW) \qquad (10a)$$

$$p_{rf}^{(+)} + p_{rf}^{(-)} = \partial_z [n(S^{(+)} - S^{(-)})] \equiv \partial_z (nS) \qquad (10b)$$

where we have taken $n \cong n^{(+)} \cong n^{(-)}$. In particular, Eq. (10b) describes the channeling of the dissipated LH power into parallel flow. Of technical interest are the total dissipated power P in front of the grill, the upstream powerflow S , and the radial width δ of the interaction region. For the old Tore Supra LH antenna [6], the surface facing the plasma is A = 0.087 m^2, its poloidal height is y_G=25 cm, $n_{edge} \cong 5\text{x}10^{17}$ m^{-3}, $(\omega_{pe}/\omega)_{edge} \cong 1.6$, $E_0 \cong 3.5$ kV/cm, and from the extent $\langle v_{ELD}, v_{max} \rangle$ of the stochastic region we estimate $n^{(hot)} \cong 0.13\, n^{(thermal)} = 6.5\text{x}10^{16}$ m-^{3}. The power flow S is obtained on integration of (10b) , for P the power density p_{rf} is integrated over the radial direction using $k_\perp$ from the LH slow wave dispersion relation, and finally δ is obtained from powerflow conservation :

$$S = \frac{2}{3}\sqrt{\frac{2}{\pi}}\,T_{//}^{3/2} \cong 33\,\text{MW/m}^2, \quad P = \frac{n^{(hot)} mA}{\pi d \omega_{pe}} v_q^2 v_{max}^2 \cong 30\,\text{kW}, \quad \delta \cong \frac{P}{S y_G} \cong 0.4\,\text{mm}$$

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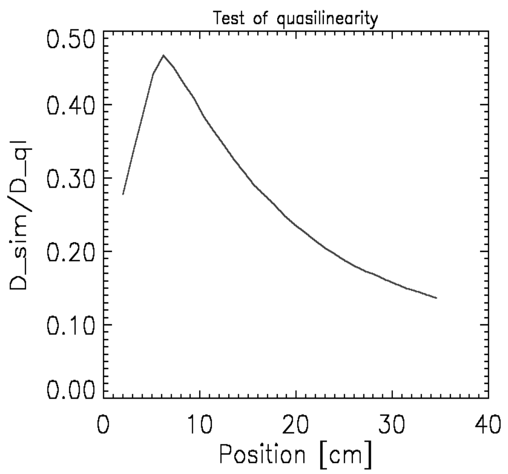

Fig. 1 The ratio of fast electron diffusion coefficients from simulation and quasi-linear theory for E_0=3.5 kV/cm.

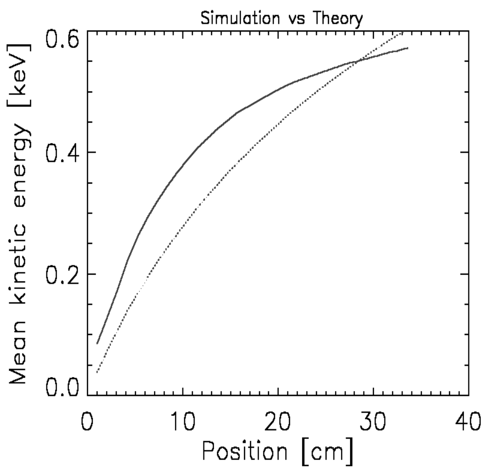

Fig. 2 The fast electron average kinetic energy from simulation (full line) and theory.