Particle diffusion in a system of magnetic islands in tokamaks in fully Hamiltonian approach

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Particle dynamics in a system of magnetic islands is studied. It is shown that fully Hamiltonian description of particle dynamics yields results different from those obtained in field lines description.

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1 Introduction

Idea of ergodic divertor awoke an interest in stochastic behavior of magnetic field lines in some magnetic systems. The first discussion of this phenomenon goes back to Rosenbluth et al. [1]. Intensive studies of ergodic divertors for Tore Supra and Textor proved the effectiveness of this method for the extraction of undesirable ions from the plasma. The method consists in the chaotization of magnetic lines which are close to the first wall, and, consequently, in the chaotization of particles, coupled to the field line system.

Chaotization is here performed by means of an artificial resonant generation of a set of mutually overlapping magnetic islands at rational surfaces. The chaotization of field lines is then a result of non–integrability of the dynamics of field lines in such magnetic systems.

It is of some interest to compare the threshold for the chaotic field lines geometry and particles behavior in such chaotic field line system. Generally, it is expected that a particle remains close to its field line. There are two reasons why particles may not follow it. Both are caused by the fact that Hamiltonian description of particles trajectories (as a more exact solution of particle dynamics) is close to the field line system only for the homogeneous magnetic field. For a non–homogeneous magnetic field, important drifts appear and force the particle to move far from the field line. (First informations appeared in [2]). Secondly, in a full Hamiltonian approach, the particle exhibits Larmor motion. This could form a nonlinear coupled system with the perturbation of the magnetic field which creates the magnetic islands.

Such coupling could possibly lead to effects which the field line description or drift approximation could not describe.

In the paper, both effects are discussed. For a set of used parameters, the effect of the drift in non–homogeneous magnetic field dominates. The effect of mutual influence of the Larmor motion and the field of magnetic islands was for our parameters not recognizable.

We consider the case when magnetic islands appear as a perturbation of the basic magnetic field of a tokamak, generated resonantly at some rational surfaces. We assume that the amplitudes of magnetic islands can be changed in some region. The basic drift is caused by the curvature of toroidal field lines. To realize the effect of this drift, we compare this dynamics with an equivalent case without the effect of the curvature (and, therefore, without this drift) in an equivalent cylindrical approximation.

First results of our study appeared in the paper [2]. There, we discussed the dynamics of particles, influenced in the tokamak magnetic field by one magnetic island. We found there that the effect of the vertical drift, caused by the curvature of the toroidal magnetic field lines is very important and changes a bit the contemporary expectations. That paper also contains a more thorough discussion and proofs of the facts that we briefly summarize in the next section.

Our present paper can be considered as a continuation of [2]. The goal is to extend the study of the dynamics of particles in the tokamak magnetic field to the case two magnetic islands, which has important qualitative differences compared to the case of one island. We also explored the range of parameters (especially the perpendicular and parallel energies) more systematically.

Our paper is organized as follows: Section 2 summarizes the formalism used, Section 3 describes the models investigated and the method used and Section 4 describes the results obtained.

2 Hamiltonian description of field lines and particles

It is a long known fact that magnetic field lines in a toroidal geometry (such as that of a tokamak) can be described by equations formally identical to the Hamilton equation of motion [3, 4]:

$$\frac{\mathrm{d}\psi}{\mathrm{d}\zeta} = -\frac{\partial F}{\partial \theta^*}; \ \frac{\mathrm{d}\theta^*}{\mathrm{d}\zeta} = \frac{\partial F}{\partial \psi}\,. \tag{1}$$

Here, the poloidal flux F represents the Hamiltonian, the intrinsic poloidal angle θ^* and the toroidal flux ψ are the conjugate position and momentum and the toroidal angle ζ has the role of time. The poloidal and toroidal fluxes can be identified with components of the vector potential: $A_{\theta^*} = \psi$ and $A_\zeta = -F$.

(For a more thorough discussion of the above stated facts we refer e.g. to the previous paper [2].)

In an equilibrium configuration, F depends only on ψ as $\frac{\mathrm{d}\psi}{\mathrm{d}F} = q$, where q is the safety factor. The equations (1) are then integrable and magnetic surfaces originate. For internal (turbulence) or external (divertors) reasons, the field can be perturbated. Such perturbation can be expressed by adding a term δF to the Hamiltonian:

$$F(\psi, \theta, \zeta) = F_0(\psi) + L\delta F(\psi, \theta^*, \zeta)\,, \tag{2}$$

L is a dimensionless parameter expressing the magnitude of the perturbation.

The added perturbation can cause nonintegrability of the Hamiltonian, with the possibility of appearance of deterministic chaos. For the study of chaos, it is convenient to decompose δF in a Fourier series:

$$\delta F(\psi, \theta^*, \zeta) = \sum_{(m,n)} \delta F_{m,n}(\psi) \cos(m\theta^* - n\zeta + \phi_{m,n})\,. \tag{3}$$

Every term in the series can resonate with a value of q equal to m/n, creating a a chain of magnetic islands. Chaotic regions usually appear first at the separatrix surface which separates the island from the magnetic surfaces around it.

For the case of magnetic surfaces with a circular cross–section, the θ^* and ψ coordinates are related to the toroidal coordinate system (ζ, θ, r) as: $\theta^*(\theta, r) = \theta - r/R_0 \sin\theta$ and $\psi(r) = B_0 r^2/2$.

A charged particle in this magnetic field moves according to a Hamiltonian

$$\begin{aligned} H(\mathbf{p}, \mathbf{q}) \quad &= \quad \frac{1}{2M}\left((p_\zeta + eF)^2 g^{\zeta\zeta} + (p_\theta - e\psi(1 - r/R_0\cos\theta))^2 g^{\theta\theta}\right) + \\ &\quad + \frac{1}{2M}\left(p_r + e\psi\frac{\sin\theta}{R_0}\right)^2\,, \end{aligned} \tag{4}$$

where (p_ζ, p_θ, p_r) are the generalized momentum conjugate to (ζ, θ, r) and g^{ii} are components of the metric tensor. The equations of motion can be obtained by differentiating the Hamiltonian. This has been done explicitly in [2].

3 Methods used for comparing field lines and particle trajectories

We integrated numerically the differential equations of the field line and the canonical equations of a charged particle. The intersections of field lines and trajectories of gyration centers of particles with a poloidal plane were plotted. The resulting figures were used for the comparison of field line and particle behavior.

The calculations were done for a model of the magnetic field with two Fourier components of the perturbation, one with $m = 3$ and $n = 1$ and the other with $m = 4$ and $n = 1$. This perturbation gives two magnetic islands, located at the value of $q = 3$ and $q = 4$, because there the field lines are resonant with the perturbation. In the poloidal cross–section, they appear as two chains of three and four island structures encircling the minor axis of the torus. The perturbation δF depends quadratically on the radial coordinate r. The corresponding perturbation of $\mathbf{B}$ then grows linearly from the minor axis towards the edge. The expression for the perturbation is then

$$\delta F(\psi, \theta^*, \zeta) = \psi\left(\cos(4\theta^* - \zeta) + \cos(3\theta^* - \zeta)\right)\,. \tag{5}$$

This perturbation was chosen to be as simple as possible, while having specific properties from the point of view of the chaos theory. It is known to lead to chaotic behavior of field lines, which manifests itself mainly in the separatrix regions of the island chains. It is also known that a transition to a global chaos can occur, where the field lines starting at one island chain can reach the area of the other island chain. This happens approximately when the perturbation is so large that the islands chains become wide enough for their stochastic separatrix regions to overlap. This is called the *Chirikov criterion.* As this result holds for field lines, we were interested if there are significant differences for particle trajectories.

In the previous paper [2], we also used a perturbation with only one Fourier component. It leads to an integrable Hamiltonian for the field lines, because a canonical transformation can transform it to a system equivalent to a pendulum. This makes such a system an interesting point for comparison of field line and particle dynamics, because the field lines do not show chaotic behavior, while for particles, the possibility of nonintegrability and chaos can not be ruled out.

The actual parameters used for the computations were: major radius $R_0 = 0.4$ m, minor radius $a = 0.1$ m, and toroidal magnetic field at the minor axis $B_0 = 1$ T. Those are the parameters of the CASTOR tokamak (see e.g. [5]). The edge value of q was 8. The particle mass and charge were chosen equal to those of a singly–ionized ion of carbon. For a numerical solution of a particle performing cyclotron motion, the time step must be sufficiently smaller than the Larmor gyration period. We chose 1/12 of the Larmor period $2\pi/\omega_c$ as the maximum integration time step. The accuracy of computation was checked by comparing the final energy to the original values. For an exact solution, they would be equal, as the particle Hamiltonian is conservative. For both field lines and particles, we obtained 20 000 intersections with the poloidal plane for every initial condition.

As a characteristic property of charged particle motion is the drift of the gyration center relative to the field lines, we were interested in estimating the contribution of this effect to differences in chaotization of field lines and particles. Because the drift is caused by the curvature of the field lines in the toroidal geometry and by the radial gradient of the magnetic field [6], we developed a cylindrical model with periodic boundary conditions which neglects the curvature. In this model, the torus is replaced by a straight cylinder. As there is no drift due to curvature and field gradient, any observed differences would be caused by other characteristics of the particle motion, such as the the cyclotron motion.

4 Results

First two figures (Fig. 1, Fig. 2) compare the dynamics of field lines and particles for two values of the stochasticity parameter L: $L = 0.0007$ and $L = 0.0025$ in the cylindrical approximation. For those values, we did not find a significant difference in chaotization, which is small for the smaller value of L and important for the larger value for both field lines and particles. This shows so far that in this approximation, which neglects the drift but not the cyclotron motion, the diffusion of the magnetic

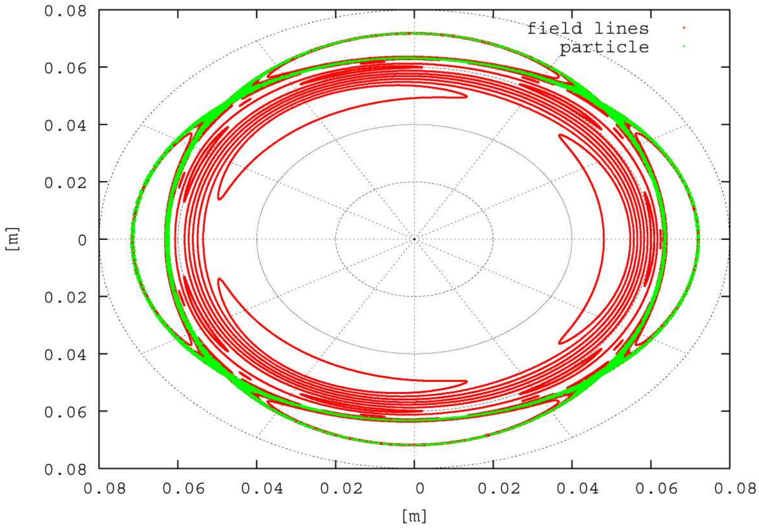

Fig. 1. Trajectory of a particle with parallel and perpendicular energies of 20 eV (green) and field lines (red) in the cylindrical approximation

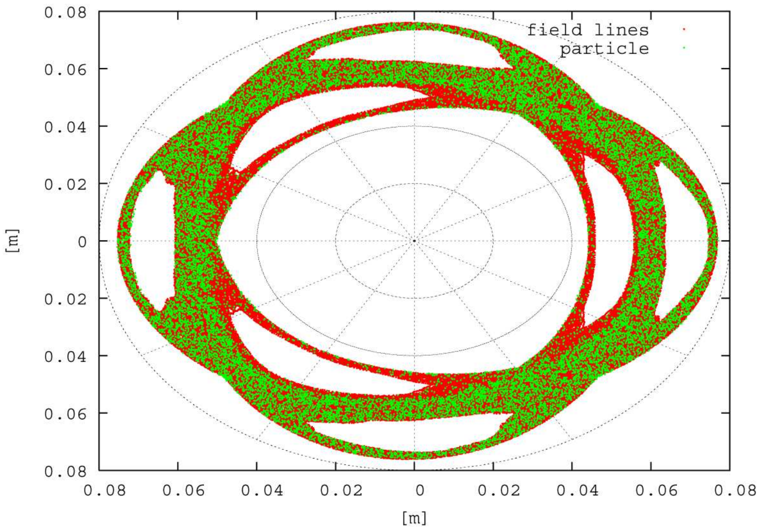

Fig. 2. Same as in Fig. 1, $L = 0.0025$

field lines is a sufficient approximation for the estimation of the diffusion of particles. The impact of the interaction of the two magnetic islands on the particle motion is here similar to the impact on field lines, at least for the parameters investigated.

The following figures (Fig. 3, Fig. 4) show the results from the toroidal model with two magnetic islands. In Fig. 3, the stochasticity of the field lines is limited

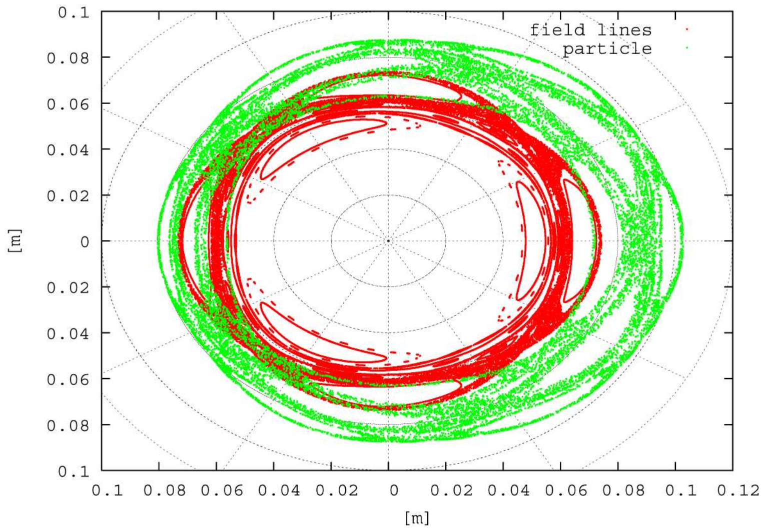

Fig. 3. Trajectory of a particle with parallel and perpendicular energies of 5 eV (green) and field lines (red) in the toroidal model, $L = 0.0014$

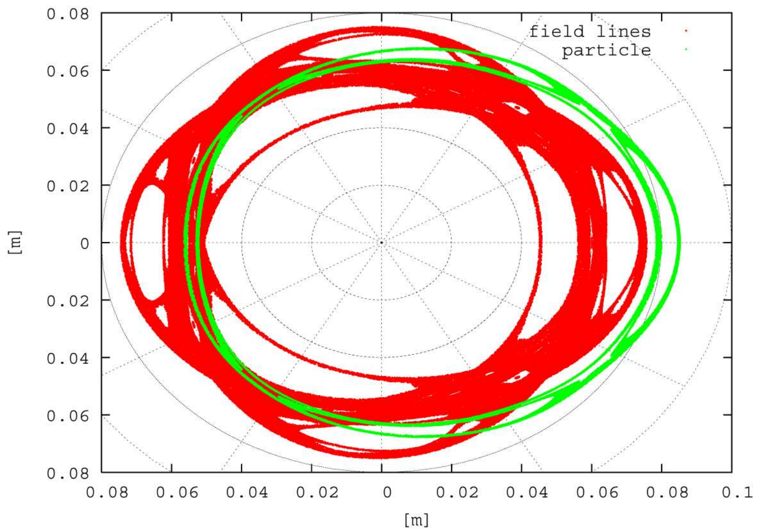

Fig. 4. Trajectories of particles with parallel energy 8 eV and perpendicular energy 16 eV (green) and field lines (red) in the toroidal model, $L = 0.002$

to the area around the separatrix, because the parameter L is not large enough for the islands to overlap. Contrary to this, the particle trajectory fills a large stochastic region which extends to the edge of torus. A similar result has been already obtained in the model with one magnetic island in [2]. In that case the field lines are not chaotic at all due to the integrability of the Hamiltonian, while the particle trajectory shows the same behavior as shown here. This shows that the cause of this effects is not the interaction of magnetic island chains. We did not find such result in the cylindrical approximation. We therefore believe that this effect is caused by the interaction of the drift with magnetic island(s).

Fig. 4 shows the impact of increased drift velocity. If the particle energy is increased, the magnitude of the drift also increases. At a certain point, the large chaotic area disappears and the particle trajectories starting near the separatrix do not diverge significantly from it. We confirmed the results for multiple initial conditions and values of L in both the one island and two islands models. An example is shown in Fig. 4. Here the parameter L is large enough for the two islands to overlap and to create a single large stochastic area of field lines, like in Fig. 2. Contrary to this, the particle motion (shown for three initial conditions near the separatrix of the outer island chain) does not show such stochasticity.

5 Summary

We found that in a cylindrical approximation (which excludes the effect of the curvature of the toroidal magnetic field lines), the stochasticity regime of field lines and particle stochasticity agree. The expected nonlinear effect of mutual influence on the dynamics of particles, trapped in one island by the field of the second island was not recognizable. The discussion of the same dynamics, but in the tokamak field, brings very interesting phenomena. Namely, in the case of moderate vertical drift, the effect of particle stochasticity clearly dominates over the effect of stochasticity of magnetic field lines. On the contrary, for large vertical drift, we found the regime of strong stochasticity of magnetic field lines, which is accompanied by a negligible stochasticity of particles. More activity in this area to examine a broader set of parameters is necessary.

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