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2010 Plasma Phys. Control. Fusion 52 085005

(http://iopscience.iop.org/0741-3335/52/8/085005)

IOP PUBLISHING PLASMA PHYSICS AND CONTROLLED FUSION

Plasma Phys. Control. Fusion **52** (2010) 085005 (13pp) doi:10.1088/0741-3335/52/8/085005

Diamagnetic measurements and plasma energy in toroidal systems

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Received 26 February 2010, in final form 1 June 2010
Published 23 June 2010
Online at stacks.iop.org/PPCF/52/085005

Abstract
The sensitivity of a diamagnetic signal to several operational and geometrical factors is analysed. Among them are the flux conservation in the plasma, eddy currents induced in the outer structures in fast processes, toroidal shift and deformation of the plasma boundary due to its energy change and inhomogeneity of the confining magnetic field. It is shown that in each case, under proper experimental circumstances, the contribution, unaccounted in the traditional theory of diamagnetic measurements, can reach a level compared with β (ratio of the volume-averaged plasma pressure to the magnetic field pressure). The approach is fully analytical with all relevant dependences shown explicitly, allowing easy estimates and suggesting a resolution of the problem in order to restore the accuracy of finding β from diamagnetic measurements. This essentially extends the analysis by Yamaguchi *et al* (2006 *Plasma Phys. Control. Fusion* **48** L73) of possible measures to improve the separation of the useful fraction of the measured diamagnetic signal. The approach is aimed at explaining the discrepancies between model estimates and experimental results, unification of a knowledge obtained in separate numerical studies, extending a theoretical basis of magnetic diagnostics and uncovering potential dangers in interpretations. This is also an essential step from traditional cylindrical theory to analytical derivations in the toroidal geometry. The results are equally applicable to tokamaks and stellarators.

1. Introduction

Diamagnetic measurements are a useful tool for determining the plasma stored energy in tokamaks and stellarators [1–9]. For interpretation a simple formula is used,

$$2\frac{\Delta\Phi}{\Phi_{\rm pl}} = \frac{B_J^2}{B_0^2} - \beta, \qquad (1)$$

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derived for a circular plasma cylinder more than 50 years ago [10] and first applied in experiments on T-5 tokamak in 1965. Here

$$\Delta\Phi = \int_{\text{loop}} (\boldsymbol{B} - \boldsymbol{B}_{\text{v}}) \cdot \text{d}\boldsymbol{S}_l, \tag{2}$$

where Φ is the flux of the magnetic field through the diamagnetic loop and $\Delta\Phi$ is the difference between the current state and initial state when $B_J = \beta = 0$, $\boldsymbol{B}$ is the magnetic field, $\boldsymbol{B}_{\text{v}}$ is the vacuum magnetic field (assumed unchanged in this case), $\Phi_{\text{pl}} = B_0 S_{\text{pl}}$ with $S_{\text{pl}} = \pi b^2$ the transverse cross-section of the plasma column, b is its minor radius, B_0 is the toroidal field, B_J is the poloidal field at the plasma boundary (the field of the net toroidal current), β is the ratio of the volume-averaged plasma pressure p to the magnetic field pressure $B_0^2/2$.

Various improvements and modifications of (1) are possible [1, 8, 9, 11–13], but (1) is sufficient for illustrating the central problem of the method: with β just several per cent or even smaller, high precision of measuring two other terms in (1) is needed. Or else, for β estimates we should look for a better relation than (1).

This was the backbone of the discussion in [9] on the accuracy of determining β from diamagnetic measurements in the Large Helical Device (LHD), the largest helical device in operation. The main subject was the effect of net toroidal current on the measurement of β, with some corrections to the net current term in (1) [9, 11, 13]. One of the outputs of that study was an estimate of the modified current term in (1) as 0.7% under some conditions. Even at the highest $\beta = 5\%$ achieved in LHD [14, 15] this 0.7% is a large quantity. If the error bars of order 10% become a practical requirement, the next step should be more precise calculation of the terms represented in (1) and a search for other possible comparable contributions to $\Delta\Phi$.

The candidates from earlier analysis [1, 12] for tokamaks are the toroidal corrections of the order of $\beta b^2/R^2$ [1] in (1), where R is the major radius, and the dependence of $\Delta\Phi$ on the plasma elongation [12]. However, the shaping effects seem unimportant for a standard LHD operation, and $\beta b^2/R^2$ gives only $\approx 0.03\beta$ for LHD parameters [9] $b = 0.6$ m, $R = 3.6$ m, which is small.

This could be a good news if not for a large difference (several times!) found in [9] between the diamagnetic β values calibrated by the new method based on 3D magnetohydrodynamic (MHD) equilibrium calculations and that from a conventional method under the cylindrical and large aspect ratio plasma model. This discrepancy remains mysterious, and it looks even more puzzling in view of earlier analytical [16] and numerical [17] predictions that, for stellarators, higher order corrections to the diamagnetic flux in (1) would be small, in qualitative agreement with the statements on the toroidal corrections in [1].

The computational results in [9] are based on reliable models and codes and are compared with experimental data. Therefore, the unaccountable difference between the approaches cannot be disregarded and should be considered as a serious challenge to theory. Also note an unexplained difference up to 20% of the plasma stored energy estimated by the diamagnetic flux measurement and the profile measurements in LHD [18].

Our goal is to explore the sensitivity of a diamagnetic signal to the plasma ability to carry the field lines frozen, transient eddy currents in the outer structures, pressure-induced plasma shift and the change of plasma boundary due to changes in β as observed in [18]. Since (1) was found [9] practically insufficient, our main target will be corrections to (1) due to the toroidal effects that cannot be treated in the cylindrical model. Also, flux conservation, an inherent attribute of ideal MHD [19] providing an additional constraint when the plasma equilibrium evolves rapidly compared with magnetic diffusion [20], was not included in the mentioned discussion of the diamagnetic measurements. Here we show that the flux conservation affects the result in a peculiar way, but does not prevent measuring β, if properly treated. Finally, the external field can slightly change with β because of the currents induced in the vessel wall or/and other metal structures [8, 9, 21]. We will show that instead of suppression of these changes or compensation, as in DIII-D [8], this can be used as a useful effect. Also, some estimates will be derived for this effect computationally treated for LHD in [9].

Our approach here is fully analytical with all relevant dependences shown explicitly. It can be applied to tokamaks and stellarators.

2. General preliminary comments

The toroidal flux linked by a diamagnetic loop can be expressed as

$$\Phi_{\rm loop} = \Phi_{\rm pl} + \Phi_{\rm gap}, \tag{3}$$

where $\Phi_{\rm pl}$ is the toroidal magnetic flux through the plasma cross-section $S_{\rm pl}$,

$$\Phi_{\rm pl} \equiv \int_{\rm plasma} \boldsymbol{B}\cdot {\rm d}\boldsymbol{S}_{\rm pl}, \tag{4}$$

and $\Phi_{\rm gap}$ is the magnetic flux through the gap between the plasma and the diamagnetic loop:

$$\Phi_{\rm gap} \equiv \int_{\rm gap} \boldsymbol{B}\cdot {\rm d}\boldsymbol{S}_{\rm g}. \tag{5}$$

Here $\Phi_{\rm pl}$ corresponds to $\Phi_{\rm dia} + \Phi_{Ip}$, and $\Phi_{\rm gap}$ to $\Phi_{\rm ext} + \Phi_{\rm vac}$ in equation (1) in [9].

The diamagnetic loop can measure only the time derivative of the magnetic flux $\Phi_{\rm loop}$, which will give at the end

$$\delta\Phi \equiv \Phi_{\rm loop}(t) - \Phi_{\rm loop}(t_0) = \int_{\rm loop} [\boldsymbol{B}(t) - \boldsymbol{B}(t_0)]\cdot {\rm d}\boldsymbol{S}_l, \tag{6}$$

where t is the observation time and t_0 the starting moment of integration. The purpose of the measurements is to get information on the final equilibrium configuration, but $\delta\Phi$ equally depends on its parameters at t_0. Antisymmetry of $\delta\Phi$ with respect to t and t_0 means that interpretation of the measurements is improved with better knowledge of some reference state at t_0. The best could be 'before the discharge' with $\boldsymbol{B}(t_0)$ the known vacuum field. Another convenient possibility is to start from t_0 when the plasma is good enough to be considered as an ideal conductor. The latter implies evolution of the magnetic field under the frozen-in constraint relating $\boldsymbol{B}(t)$ to $\boldsymbol{B}(t_0)$.

Such a situation is typical for experiments with a fast increase in β when the heating power rapidly increases, as, for example, in [3–5, 7, 9]. In theory the frozen-in constraints are incorporated into the analysis of flux-conserving tokamaks [20, 22] and stellarators [13]. Flux conservation is an important part of disruption studies in tokamaks, including ITER [23]. There is one more area where the flux conservation concept should be applied: the edge cooling experiments, see [24–27] and references therein. In these experiments abrupt (or ballistic) response of the plasma to edge cooling is observed with an increase in the core temperature on a time scale of 10–20 ms. This phenomenon is called puzzling [24], lacking physical explanation [26, 27], unpredicted with proper accuracy by the existing models [25], a transport enigma [24] still unresolved. Diamagnetic measurements can be a useful diagnostic of the plasma energy changes in such experiments.

In these and similar cases evolution of the plasma parameters occurs on the time scale much smaller than its resistive time, and the plasma can be considered as a perfect conductor.

This means frozen-in magnetic field with the integral consequence $\Phi_{pl} = \text{const}$. Then a diamagnetic loop can measure only a variation

$$\delta\Phi_{gap} \equiv \Phi_{gap}(t) - \Phi_{gap}(t_0) = S_{gap}\delta B_e - B_e\delta S_{pl}, \tag{7}$$

where we represented Φ_{gap} as $\Phi_{gap} = B_e S_{gap}$ with $S_{gap} = S_{loop} - S_{pl}$, S_{loop} the surface covered by the loop, S_{pl} the surface covered by the plasma and B_e the external vacuum toroidal field averaged over S_{gap}.

A question arises—what information on the plasma can be extracted from the measurements producing a combination given by (7)? The goal of the measurements is the evaluation of the plasma stored energy, so we have to know in what way $\delta\Phi_{gap}$ may be related to β. The equilibrium equations alone are not sufficient for the answer. The key to the solution is a nontrivial modification of the boundary conditions.

3. Flux-conserving plasma evolution and diamagnetic measurements

According to (3) and (7), if Φ_{pl} is frozen, the measured diamagnetic flux must be $\delta\Phi_{gap}$, a function of δB_e and δS_{pl}. This apparent fact, a trivial consequence of definition (5), is strangely ignored in theory of diamagnetic measurements and discussions.

With $\delta\Phi_{gap}$ as an only available quantity we have to find proper dependences of δB_e and δS_{pl} on the plasma characteristics. In tokamaks and stellarators the external field can be controlled to some extent [8, 9, 28], and δB_e can be partly determined by this control. In principle, δB_e can be measured by using local probes outside the plasma or the diamagnetic double loops [6]. The double loops give also two values of $\delta\Phi_{gap}$ with different S_{gap}, which could allow one to find δS_{pl}. This can also be done when several different loops are used as described in [8]. In any case, the measurements can finally give us δS_{pl}.

This sounds unusual since such a deformation of the plasma cross-section never manifested itself as an easily visible equilibrium effect in experiments and was never discussed as dominant in diamagnetic measurements. It is well known that the plasma shape is very sensitive to the plasma parameters, and shape control is an important part of tokamak operation [8, 28]. Sophisticated methods and techniques are developed for tokamaks to minimize the difference between the plasma boundary and the desired shape described by a large set of coordinates. For example, the control system designed for ITER maintains the specified plasma current, the position and the shape in spite of slow evolution of plasma parameters, rapid changes in the additional heating and non-inductive current drive and other disturbances [28]. But in all these cases the goal is to suppress any deviation from the target.

In addition, the problem of δS_{pl} detection as a quantity related to diamagnetic measurements was never addressed in theory (except several papers, see [21, 32]) and experiments. However, recent studies [18] of the LHD plasma boundary change with β have demonstrated that the related δS_{pl} (though of a different origin from that we discuss in this section) can be large enough to be detected. The goal of that study was to establish the identification method for consistent shape and location of a plasma boundary with experimental measurements based on numerical calculations of MHD equilibrium. Also, the results [18] prove the importance of a careful definition of the plasma boundary and that disregard of its evolution leads to incorrect interpretation of the equilibrium data.

This aspect was earlier emphasized in [17] with a numerical demonstration that a change in the volume and the position of the plasma column in LHD strongly affects the signals of the magnetic field pick-up coils. In [29] substantial impact of plasma boundary modulation on LHD plasma stability was shown. Flux conservation was assumed in the numerical determination of the plasma–vacuum boundary [29], which is related to δS_{pl} in (7), but different from volume variations in [17, 18]. There is only one step from [17, 18, 29] to numerical and experimental studies of the flux-conserving effects in stellarators with the use of magnetic diagnostics. Note that change in the plasma boundary shape can be substantial in a low shear helical system even at a relatively low β [30, 31].

Analytical estimation of $\delta S_{\rm pl}$ that appears in (7) was discussed in [32]. Some integral properties of equilibrium configurations with frozen magnetic flux were also discussed in [21]. Here we show some estimates in a cylindrical approximation, based on previous results.

Conservation of the toroidal magnetic flux in the plasma means that

$$\delta\Phi_{\rm pl} \equiv \delta \int_{S_{\rm pl}} \boldsymbol{B}\cdot {\rm d}\boldsymbol{S}_{\rm pl} = 0, \tag{8}$$

which can be written as

$$S_{\rm pl}\delta\bar{B}_z + \bar{B}_z\delta S_{\rm pl} = 0 \tag{9}$$

with $\Phi_{\rm pl} = \bar{B}_z S_{\rm pl}$ and

$$\delta\bar{B}_z \equiv \bar{B}_z\big|_{\rm before}^{\rm after} \tag{10}$$

calculated as a difference in the final and initial states. In (9) B_z stands for the toroidal field, which is the field along the vertical axis z in the cylindrical model, the bar denotes the averaging over the total cross-section of the plasma column:

$$\bar{f} \equiv \frac{1}{S_{\rm pl}}\int f\,{\rm d}S_{\rm pl} = \frac{2}{b^2}\int_0^b fr\,{\rm d}r. \tag{11}$$

The second equality here is for the circular plasma of minor radius b.

In (9) we need $\delta\bar{B}_z$ which can be found from

$$\bar{B}_z = B_{\rm e}\left(1 - \frac{\beta}{2} + \frac{B_J^2}{2B_{\rm e}^2}\right), \tag{12}$$

which is a consequence of the equilibrium equation in cylindrical approximation, see, for example, [1, 19, 21, 33] for details. Here $\beta \equiv 2\bar{p}/B_{\rm e}^2$ and $B_J = B_\theta(b)$ is the poloidal field due to the net plasma current, the same as in (1). With this formula and (9) we obtain from (7)

$$\delta\Phi_{\rm gap} = S_{\rm loop}\delta B_{\rm e} + \delta\Phi_{\rm pl}^0, \tag{13}$$

where

$$\delta\Phi_{\rm pl}^0 \equiv \frac{B_{\rm e}S_{\rm pl}}{2}\left(\frac{\delta B_J^2}{B_{\rm e}^2} - \delta\beta\right) \tag{14}$$

can be called the variation of the toroidal flux in the plasma when $\delta B_{\rm e} = \delta S_{\rm pl} = 0$. Tokamaks and stellarators are the systems with small β (just several per cent). Accordingly, variations $\delta B_{\rm e}$ and $\delta\bar{B}_z$ are small, and we keep here only the linear terms in β.

Let us recall that (13) is obtained under condition $\Phi_{\rm pl} = {\rm const}$, so that $\delta\Phi_{\rm loop} = \delta\Phi_{\rm gap}$. If $\delta B_{\rm e} = 0$, which can be provided by the control of the toroidal field or special compensation [8] of $\delta B_{\rm e}$, equation (13) gives exactly the signal expected for equilibrium plasma when its deformation $\delta S_{\rm pl}$ and flux conservation $\Phi_{\rm pl} = {\rm const}$ are both disregarded. That is why determining β by measuring $\delta\Phi$ is possible even when the magnetic flux is frozen into the plasma, with proper calibrations allowing separation of the hindering term $S_{\rm loop}\delta B_{\rm e}$ in (13).

Let us emphasize once more that flux conservation in the plasma, $\Phi_{\rm pl} = {\rm const}$, does not prevent reliable measurement of $\delta\beta$ only because the plasma is properly deformed with β change, though this deformation is so small that it was never experimentally identified.

In this context, this is a useful property. On the other hand, it demonstrates high sensitivity of the diamagnetic signal to the plasma shape. Under different circumstances this was also found in numerical calculations of two realistic free-boundary equilibrium sequences for the LHD [17]. Below those results will be commented when other deformations of S_{pl} will be discussed. Before that we give analytical estimates for the first term in (13) when δB_e appears as a response of the outer conductors to the magnetic perturbation from the plasma.

4. Ideal wall and diamagnetic measurements

The term $S_{loop}\delta B_e$ corresponds to Φ_{ext} in [9]. According to [9], in LHD the field B_e is changed during the discharge because of the eddy currents in external coils such as helical and poloidal coils and in structures such as the vacuum vessel and supporting shells. The contribution due to δB_e, whose evaluation was a part of the precise numerical calculations in [9], was found to be a noticeable fraction of the measured diamagnetic signal in LHD. Here we present a simple analytical estimate for δB_e.

Assume that the plasma is surrounded with an ideal wall preventing the penetration of the magnetic flux. Such a situation was described as requiring special measures in measuring the diamagnetic signal also in DIII-D [8]. In this case the total magnetic flux

$$\Phi_w = \Phi_{pl} + \Phi_{out} \tag{15}$$

through the cross-section S_w of the vessel must be conserved. Here, as introduced above, Φ_{pl} is the magnetic flux through the plasma cross section S_{pl}, and $\Phi_{out} = B_e(S_w - S_{pl})$ is the flux of the field B_e in the plasma–wall vacuum gap $S_w - S_{pl}$.

When the magnetic flux in the plasma is frozen-in ($\delta\Phi_{pl} = 0$), the conservation of Φ_w means $\Phi_{out} = \text{const}$ or

$$(S_w - S_{pl})\delta B_e - B_e\delta S_{pl} = 0, \tag{16}$$

which gives us

$$\delta B_e = \frac{B_e\delta S_{pl}}{S_w - S_{pl}} = -\frac{S_{pl}}{S_w - S_{pl}}\delta\bar{B}_z. \tag{17}$$

In the last equality we used equation (9) to express δS_{pl} and again disregarded the difference between $\bar{B}_z$ and B_e, see (12) with $\beta \ll 1$ and $B_J^2/B_e^2 \ll 1$. Equation (12) also implies that

$$\delta B_e = \delta\bar{B}_z - \frac{\delta\Phi_{pl}^0}{S_{pl}}, \tag{18}$$

where $\delta\Phi_{pl}^0$ is defined by (14), and finally we have

$$\delta\bar{B}_z = \left(1 - \frac{S_{pl}}{S_w}\right)\frac{\delta\Phi_{pl}^0}{S_{pl}}. \tag{19}$$

Then

$$\delta B_e = -\frac{\delta\Phi_{pl}^0}{S_w}, \tag{20}$$

and combining (13) and (20) we obtain

$$\delta\Phi_{gap} = \alpha\delta\Phi_{pl}^0 \tag{21}$$

with $\alpha = \alpha_{id}$ if the wall is considered a perfect conductor:

$$\alpha_{id} \equiv 1 - S_{loop}/S_w. \tag{22}$$

If the wall is not 'ideal', the contribution $S_{loop}\delta B_e$ to (13) due to the plasma-induced δB_e will be smaller, so that

$$\alpha_{id} \leqslant \alpha \leqslant 1, \tag{23}$$

where the lower limit comes from (22), and the upper limit corresponds to the non-conducting wall or $\delta B_e = 0$ in (13).

Expression (21) with $\alpha = \alpha_{id}$ shows two important things. First, the only contribution to the diamagnetic signal in the flux-conserving case, $\delta\Phi_{gap}$, depends on the loop size, as implied by (22). Second, with $\alpha \neq 0$ the measured $\delta\Phi_{gap}$ is proportional to $\delta\Phi^0_{pl}$.

Equation (22) gives $\alpha_{id} = 0$ for a loop just near the wall and, accordingly, $\delta\Phi_{gap} = 0$. This exactly reproduces the situation observed in the DIII-D tokamak with a loop originally installed just on the inner side of the wall [8]. That loop has been removed with a motivation that DIII-D vacuum vessel acts as a flux conserver on short time scales so that the diamagnetic loop on the interior surface has no significant advantage in terms of time response [8]. We can argue that such a loop could be useful on the longer time interval. Also, it could be used in pair with some other loop for a direct measurement of δB_e.

Proportionality of $\delta\Phi_{gap}$ to $\delta\Phi^0_{pl}$ defined by (14) means that, even with flux conservation in the plasma and, maybe, inside the wall, the diamagnetic signal remains a function of β similar to (1), only with a simple geometrical coefficient (22).

5. Diamagnetic signal and Shafranov shift

Dependence of the diamagnetic signal on the Shafranov shift was demonstrated analytically in [16] and briefly discussed in [34]. This can be illustrated by a simple formula:

$$\delta\Phi_{pl} \equiv \frac{1}{2\pi}\int_{plasma} \frac{F - F_e}{r^2}\,dV, \tag{24}$$

where the plasma shape deformation is disregarded. This formula, approximate for stellarators and precise for tokamaks, comes from the definition

$$\Phi_{pl} = \frac{1}{2\pi}\int_{plasma} \boldsymbol{B}\cdot\nabla\zeta\,dV \tag{25}$$

of the total toroidal magnetic flux through the plasma column and the conventional expression $\boldsymbol{B}_t = F\nabla\zeta$ for the toroidal magnetic field. In (25) the integration is performed over the plasma volume, ζ is the geometrical toroidal angle in the cylindrical coordinates r, ζ, z related to the main axis and F_e is F at the plasma boundary. The region near the magnetic axis, where the difference $F - F_e$ must be maximal, gives smaller contribution into (24) when the axis is shifted to larger radius r, which is the effect we discuss here.

The calculation for a current-free equilibrium plasma with F_e the same at t and t_0 finally gives [16]

$$\delta\Phi_{pl} = -\frac{1-\delta_{Sh}}{2\pi F_e}\int_{plasma} p\,dV, \tag{26}$$

where δ_{Sh} is a quantity related to the Shafranov shift. It can be estimated as [16]

$$\delta_{Sh} = C\frac{b}{R}\frac{\Delta_{ax}}{b}, \tag{27}$$

where Δ_{ax} is the shift of the magnetic axis, b is the minor radius of the plasma, R is its major radius and C is the constant of order unity depending on the pressure distribution.

With (26) equation (1) should be modified to the form ($B_J = 0$ here)

$$\beta = -\frac{2}{1-\delta_{Sh}}\frac{\Delta\Phi}{\Phi_{pl}}. \tag{28}$$

It is clear that δ_{Sh} is a measure of inaccuracy in determining β when the Shafranov shift is ignored. For LHD parameters [9] $b = 0.6$ m, $R = 3.6$ m and $\Delta_{ax}/b < 0.5$ equation (27) gives $\delta_{\rm Sh} < 0.1$, which is small even for very large $\Delta_{\rm ax}$. This justifies the use of the cylindrical formulae like (1) instead of the toroidal results. Equations (26) and (28) show that disregarding the Shafranov shift in the diamagnetic measurements results in a slightly underestimated β.

6. Plasma global shift and diamagnetic signal

In tokamaks and stellarators the β rise leads also to the outward shift of the plasma as a whole, which can be called the global shift Δ_b. It can be affected by the external vertical field $B_\perp$ so that $\Delta_b = \Delta_\beta + \Delta_\perp$, where Δ_β is the pressure-induced shift at $B_\perp = 0$, and $\Delta_\perp$ is due to $B_\perp$. In tokamaks the shift Δ_b is dynamically suppressed by the equilibrium control systems [8, 28]. In stellarators such a control is not vitally important, though theory predicted [35], experiments proved [3] and calculations confirmed [17] that it can considerably facilitate high-β operation.

Estimates [32, 36, 37] for conventional stellarators show that the pressure-induced outward shift Δ_β of the plasma column must be fairly large at high β:

$$\frac{\beta}{2\beta_{\rm eq}^0} \leqslant \frac{\Delta_\beta}{b} \leqslant \frac{\beta_0}{2\beta_{\rm eq}^0}. \tag{29}$$

Here $\beta_{\rm eq}^0 = \mu_b^2 b/R$, μ_b is the rotational transform at the plasma edge, b is the averaged minor radius of the plasma, R is the major radius, B_0 the toroidal magnetic field at $r = R$ and $\beta_0 = 2p(0)/B_0^2$ is the local β value at the magnetic axis.

For LHD with $\mu_b \approx 1$ and $b/R \approx 0.15$ we have $\beta_{\rm eq}^0 = 0.15$, and the lower bound in (29) is 0.1 for $\beta = 3\%$, which corresponds to Δ_β larger than 6 cm. Note that $\beta = 5\%$ has already been achieved in LHD [14, 15], which means shift Δ_β larger than 10 cm. An outward shift of the same order was indeed experimentally found in LHD and numerically reproduced [18]. Note that LHD is a flexible device allowing operation with the axis position $R_{\rm ax}$ of the vacuum configuration varying from 3.4 to 4.1 m [14]. Inward and outward shifts in this range can essentially reduce or increase the LHD configuration sensitivity to β [38]. Then smaller or larger Δ_β are possible than implied by (29), which is an estimate.

The global shift Δ_b can be partially suppressed by the field $B_\perp$ generated due to the eddy currents in external conductors. For the plasma surrounded with an ideal wall of radius a_c this results in [37]

$$\Delta_b = \Delta_\beta(1 - b^2/a_c^2). \tag{30}$$

For LHD, a rough estimate is $b/a_c = 0.6/0.9$ [39], so that $\Delta_b > \Delta_\beta/2$ which, as explained above, can be several centimetres. Note that Δ_b of order 5–6 cm has been found in [18].

When the plasma is globally shifted outwards, it moves from a stronger to a weaker toroidal field. Then, in addition to the effects described by (7), the combined effect of the plasma boundary deformation and $1/r$ dependence of the vacuum toroidal field should be taken into account.

In the axially symmetric toroidal geometry, which can be considered for illustrations and first estimates, the flux in the plasma-loop gap is given by

$$\Phi_{\rm gap} \equiv \int_{\rm gap} \boldsymbol{B}\cdot {\rm d}\boldsymbol{S}_g = \frac{1}{2\pi}\int_{p-g} \boldsymbol{B}\cdot\nabla\zeta\,{\rm d}V = \frac{F_{\rm e}}{2\pi}\int_{p-g}\frac{{\rm d}V}{r^2}, \tag{31}$$

where $p - g$ means the volume between the plasma and (axisymmetric) toroidal surface with the diamagnetic loop in the cross-section $\zeta = $ const. Using the standard formula

$$\frac{\rm d}{{\rm d}t}\int_V f\,{\rm d}V = \int_V \frac{\partial f}{\partial t}{\rm d}V + \oint_S f\boldsymbol{v}\cdot{\rm d}\boldsymbol{S}, \tag{32}$$

where $\boldsymbol{v}$ is the velocity of the boundary $\boldsymbol{S}$ of this volume, and the integral with $\boldsymbol{v}$ is taken over this boundary, we obtain

$$\frac{d\Phi_{gap}}{dt} = \frac{\Phi_{gap}}{F_e}\frac{\partial F_e}{\partial t} + \frac{F_e}{2\pi}\oint \frac{\boldsymbol{v}\cdot d\boldsymbol{S}}{r^2}. \quad (33)$$

Since the diamagnetic loop does not move, this is reduced to

$$\frac{d\Phi_{gap}}{dt} = \frac{\Phi_{gap}}{F_e}\frac{\partial F_e}{\partial t} - \frac{F_e}{2\pi}\oint_{plasma} \frac{\boldsymbol{v}\cdot d\boldsymbol{S}_n}{r^2}, \quad (34)$$

where $\boldsymbol{S}_n$ means the boundary surface of the plasma, oriented outwards.

Here the first term can be identified as finally generating $S_{gap}\delta B_e$ in (7), which can be applied to toroidal configurations with a proper definition of B_e. The second, after time integration, will give us $-B_e\delta S_{pl}$ if the toroidal corrections are disregarded (cylindrical or large-aspect-ratio approximation). However, strict definition in toroidal geometry,

$$\frac{dS_{pl}}{dt} = \frac{1}{2\pi}\oint_{plasma} \frac{\boldsymbol{v}\cdot d\boldsymbol{S}_n}{r}, \quad (35)$$

implies that this is not a complete answer. It is clear that velocity $\boldsymbol{v}_0 = \nabla U(r,z)\times \boldsymbol{e}_\zeta$ gives zero in (35) or $\delta S_{pl} = 0$, but does not nullify the last term in (34). Here r, ζ, z are the same cylindrical coordinates as introduced in the previous section, $\boldsymbol{e}_i$ are the unit vectors along proper axes. With $U = -v_r z e_r$ we have $\boldsymbol{v}_0 = v_r\boldsymbol{e}_r$, if v_r is constant, and

$$\oint_{plasma} \frac{\boldsymbol{v}_0\cdot d\boldsymbol{S}_n}{r^2} = -v_r\int_{plasma}\frac{dV}{r^3} \approx -2\pi\frac{\Phi_{pl}}{F_e}\frac{v_r}{R}. \quad (36)$$

With this relation we obtain from (34) a toroidal modification of (7):

$$\delta\Phi_{gap} = S_{gap}\delta B_e - B_e\delta S_{pl} + \Phi_{pl}\frac{\delta\Delta_b}{R}, \quad (37)$$

where $\delta\Delta_b$ is the increment of Δ_b, and Φ_{pl} is the total toroidal flux (25) in the plasma. The last term here describes the result of the toroidal shift of the boundary in the toroidal geometry, while δS_{pl} is determined by $\boldsymbol{v}-\boldsymbol{v}_0$ (here, by definition, $\boldsymbol{v}_0$ gives $\delta S_{pl} = 0$). This term is positive for the outward shift, which is natural since with $\delta\Delta_b > 0$ the plasma comes to a region of weaker toroidal field, and the area with larger field covered by the loop becomes larger. This is illustrated in figure 1 where $\delta\Delta_b > 0$ corresponds to the evolution from the left (solid) to the right (dashed).

In tokamaks the plasma global shift was routinely measured long ago [1, 2, 33]. In stellarators this is also possible [4, 18, 32, 40]. Such measurements can be useful combined with (37) and diamagnetic measurements.

In [1] it was stated that a relative error of order Δ/R appears when B_0 in $\Phi_{pl} = B_0 S_{pl}$ in (1) is a field at the centre of the vacuum vessel, but the plasma centre is shifted by Δ. This was obtained as an estimate for a circular tokamak by some expansion. Our relations (34) and (37) are more general and not related to (1) in any way. Actually these formulae are for the flux of the vacuum field *outside* the plasma, while (1) describes the flux *in* the plasma. It is important that, when the magnetic flux is frozen, $\delta\Phi_{gap}$ given by (37) will be an essential part of the measured diamagnetic signal.

Expression (37) shows that $\delta\Delta_b = 6$ cm in LHD configuration with $R = 3.9$ m will give 3% to $2\delta\Phi_{gap}/\Phi_{pl}$. This should be compared with β, as implied by (1). This contribution is certainly above the level of 0.7% found in [9] as an essential quantity for proper calibration of the diamagnetic measurements in LHD.

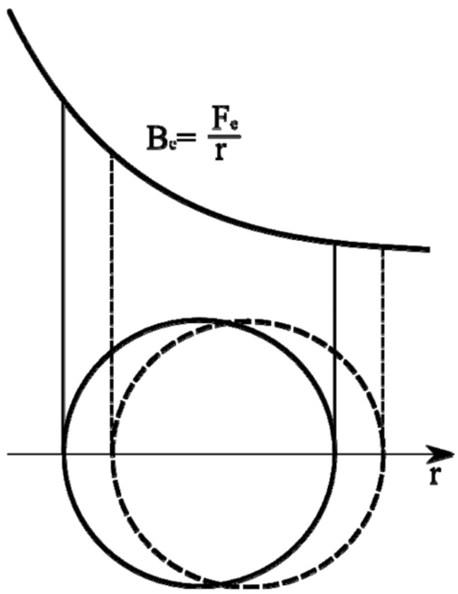

Figure 1. Two positions of the plasma and the external toroidal field $B_e = F_e/r$. Larger radial shift of the plasma brings it to a weaker toroidal field.

This shows that the effect related to $1/r$ radial dependence of the toroidal field and the pressure-induced toroidal shift of the plasma boundary, not included into the cylindrical model, can give an unaccounted contribution to the measured diamagnetic signal comparable to the signal itself. Note that this estimate is rather conservative. With $R = 3.6$ m and yet realistic $\delta\Delta_b = 9$ cm we obtain for the last term in (37) $2\delta\Phi_{gap}/\Phi_b = 5\%$. This means 100% of the 'cylindrical' main term at $\beta = 5\%$.

Finally, we cannot rule out the global shift of the plasma as a possible reason for large contribution to $\delta\Phi_{gap}$, if $\delta\Delta_b$ is not suppressed.

7. Plasma volume change and diamagnetic signal. Implications to LHD

The parameter $\delta\Delta_b$ introduced in (37) is a shift that preserves S_{pl}. However, in stellarators the plasma shifting outwards enters the region with stochastic magnetic field lines [17, 18, 29]. This leads to a noticeable change in the plasma volume, which makes δS_{pl} a function of $\delta\Delta_b$. The importance of this effect for LHD was emphasized recently in [14, 18, 29]. The effect was known long ago [31], but up to now there have been no systematic methods to precisely describe the shape and location of the plasma boundary in helical systems [18, 29]. Here we can only make estimates of such boundary modulation on $\delta\Phi$ by an order of magnitude.

Equation (34) describes the dependence of Φ_{gap} on arbitrary changes in the plasma geometry, which are represented by two 'geometrical' terms in (37). The pressure-induced change in plasma volume is described by $-B_e\delta S_{pl}$ in (37). To complete the task, we have to evaluate this contribution. Precise calculations for LHD could be done with (34) if the plasma boundary change found in [18] would be transformed into local velocities $\boldsymbol{v}$ of the boundary elements.

Such a change in S_{pl} should be treated as a separate important element affecting the diamagnetic measurements. The reason is that the flux conservation discussed above leads to a small increase in the plasma volume as $\delta V_{pl}/V_{pl} \approx \beta/2$ [21, 32], see (9) and (12), while the results [18] show a much larger volume increase with an increase in β, up to 10%. It was emphasized [18] that, in such cases, the difference in the plasma stored energy estimated by the diamagnetic flux measurement and the profile measurements is up to 20%.

The above analysis shows the ways of resolving the problem. First, we can conclude that in these cases the plasma evolution cannot be flux-conserving. This is due to the mentioned difference (order of magnitude) in values $\delta S_{pl}/S_{pl}$ expected from flux conservation and observed in LHD [18]. If so, evolution of S_{pl} should be prescribed using experimental results from [18], if there is no reliable theory.

The definition of plasma boundary in stellarators is a disputed issue [17, 18]. For our purposes, a natural step could be the identification of such a peripheral closed magnetic surface which keeps Φ_{pl} = const inside this surface when the plasma equilibrium is evolved. Then the outer region can be considered as the discussed plasma-loop gap, and (37) can be used for the estimates.

This, probably, can reconcile $\Delta S_{pl}/S_{pl} \approx 0.1$, equivalent to a 10% volume increase in LHD [18], with $\delta S_{pl}/S_{pl} \approx \beta/2$ found earlier as a measure of the cross-section change in the flux-conserving case. In the first expression we have ΔS_{pl}, most certainly, related to the surfaces with different fluxes (a concept used in [17]), which is not δS_{pl} for the same flux. Also, δS_{pl} should be calculated taking account of the toroidal effects, which, as shown in the previous section, must be important. Finally, note that at $\delta S_{pl} > 0$, as we expect for the flux-conserving case and which corresponds to observations [18] (but contradicts to modelling in [17]), the two last terms in (37) are of opposite signs. If so, some partial compensation can occur, and diamagnetic measurements will remain sufficiently reliable, even at a 10% increase in the plasma volume in LHD [18].

8. Conclusion

High accuracy required for measurements of the plasma stored energy and β [1–9] calls for a better expression than (1), a more careful description of equilibrium configuration and its evolution, and even a more detailed separation of different contributions to the measured signal than has been done in [9]. To some extent this might compromise the method whose main advantage was the simplicity expressed by (1). Our analysis and estimates show that, despite the computational complications, the simplicity remains a virtue if the variations of the plasma shape and position are properly treated with due account of the toroidal geometry and boundary conditions.

A contribution of these geometrical effects to the measured diamagnetic signal can be, under certain conditions, of the order of the main term, which is β in (1). This is shown, in particular, by (21) which explains that β can be found from diamagnetic measurements even when the magnetic flux is frozen into the plasma. This formula also explicitly shows the dependence of the measured flux on the flux loop geometry (essentially different from the effect discussed in [9]) and on the boundary conditions at the wall.

Our analysis confirms that, with proper treatment of the terms, equation (1) can be used as a good basis for data analysis. However, as seen from (37), the toroidal effects, which could not be introduced by simple modifications of the 'cylindrical' quantities in (1), can give another contribution to $\delta\Phi$ comparable to β, if $\delta\Delta_b$ is sufficiently large. This undesired part can be eliminated by suppressing the global plasma shift, as routinely done in tokamaks. Or, otherwise, experimental verification of this prediction can be proposed for stellarators, which can be done by controlling the inward/outward shift of the plasma.

The plasma shift in the inhomogeneous magnetic field, with related deformations of the boundary, should be considered as the main threat to the accuracy of the diamagnetic evaluation of plasma β. This is because the plasma energy is much smaller (by a factor of β) than the energy of the confining magnetic field. Diamagnetic loops react to small redistributions of the magnetic energy which can or cannot be directly related to the perpendicular force balance responsible for the β term in (1). Therefore more detailed studies along the lines described here are needed to increase the reliability of interpretation of diamagnetic measurements.

Here, for illustration, we used equation (1) which is also valid for anisotropic plasmas if β is replaced by $\beta_{\perp}$, representing the perpendicular pressure. It is important that the geometrical effects related to the plasma boundary change and the toroidal effects do not depend on a particular model of the plasma. Therefore, we can easily combine the 'geometrical' part of the presented theory with any alternative approach to the plasma equilibrium.

Acknowledgments

The author is grateful to K Y Watanabe for discussions of experimental and computational results, to K Matsuoka, S Okamura, H Yamada and K Yamazaki for support during many years of collaboration and to P B Aleynikov for technical help.

This work was supported by the National Institute for Fusion Science, Japan. Hospitality of this Institute is greatly appreciated.

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