Journal of Fusion Energy (2022)41:17
https://doi.org/10.1007/s10894-022-00329-5

ORIGINAL RESEARCH

Magnetohydrodynamic Mode Identification for Golem Mirnov Coil Signals Using Singular Value Decomposition and Multichannel Variational Mode Decomposition Method for Analyzing Time–Frequency

Jayakumar Chandrasekaran[1] · Sangeetha Jayaraman[2]

Accepted: 30 November 2021

Abstract

In this paper, we have investigated the method to study non-stationary signal characteristics in plasma tokamak using the combination of Multichannel Variational Mode Decomposition (MVMD) and Singular Value Decomposition (SVD). We have applied this technique directly without any signal preprocessing techniques over the Mirnov coil signals to analyze the magnetic fluctuations produced by the rotating magnetic fields of the plasma in tokamaks. Extraction of Principal axes (PA) and Principal Components (PC) of multichannel Mirnov coil signals are through the singular value decomposition technique. The Multichannel variational mode decomposition technique is provided with a PC matrix to identify the dominant harmonics as K-modes. Finally, the Time–frequency analysis is carried out using Hilbert Transform (HT). The proposed technique handles multichannel Mirnov coil signals in parallel to frequency identification, and also to understand the poloidal structure during current perturbation. Artificially simulated data and Mirnov coil signals from Golem Tokamak aided in testing the proposed technique. In Golem data during the present rise phase, transition happens in the current perturbation from m = 4, poloidal structures to m = 3, and m = 2. The simulated data and Golem tokamak data generated the results of the proposed model. The article also compared this with other existing signal decomposition techniques.

Keywords Magnetohydrodynamic mode identification · Golem tokomak · Singular value decomposition · Multichannel variational mode decomposition · Time–frequency

Introduction

Investigation of the structure and behaviour of Magneto-Hydro-Dynamic (MHD) activities in tokamaks proves to be a vital and exciting niche in plasma physics. Identification and understanding of MHD activity through mode numbers provide great insights on plasma instabilities created due to magnetic islands and plasma cross cross-section.

✉ Sangeetha Jayaraman
sangita.sudhakar@gmail.com

[1] School of Computing, SASTRA Deemed to Be University, Thirumalaisamudram, Thanjavur, Tamilnadu 613401, India

[2] Department of Computer Science & Engineering, Srinivasa Ramanujan Center SASTRA Deemed University, Kumbakonam, Tamilnadu 612001, India

In tokamaks, the primary cause of plasma disruption is MHD instabilities that develop over the resonant magnetic surfaces when the value of the safety factor (q) is altered [1, 2]. The measured toroidal and poloidal mode numbers (m, n) of the magnetic field fluctuations of the rotating magnetic islands with Mirnov coils positioned on the Tokamak vacuum vessel. Spatially distributed measurements from the magnetic field perturbations provide knowledge about the coherent spatial structure. The modes' time of the evolution is obtained using the above. It helped in identifying disruption precursors and developing remediation strategies. The simple and easy-to-use design of Mirnov coils aids researchers in their plasma edge studies. The fluctuations produced by the rotating magnetic fields of the plasma in tokamaks exhibit a variety of intrinsic non-stationary and non-linear activities. Further, this necessitates the application of a non-stationary signal analysis

Published online: 09 August 2022

technique to calculate the phase, frequency, and amplitude of these activities.

In general, inference of spatial and temporal structures of MHD modes is done through spectral techniques. Then, with the application of the SVD technique over the time–space matrix of multichannel Mirnov coils principal axes that contain the mode numbers and singular values are extracted. The mode structure could be visualized by plotting the probe position against the principal axes using a polar plot. Using this second technique or more modes comprising various frequencies are visualized as a single-mode distorted in time. Two or more additional modes with the same number but different frequencies are referred to as mode coupling. It is visualized as a non-sinusoidal distorted eigenvector in space.

For analyzing time–frequency [3] in MHD modes, and disruptions in tokamak plasmas, the fusion plasma signals utilized Choi–William's distribution. Various analysis has been performed in JET tokamak on the precursors of edge localized modes, tearing modes, and washboard modes. However, the performance of Choi–William's distribution is good only when provided with the σ parameters, the choice of window length of signal components to avoid artefacts. Hence, the distribution fails in self-adaptive nature.

The most widely used technique in the time–frequency analysis of non-linear signals is Empirical Mode Decomposition (EMD) [4–7]. In EMD, a multi-constituent signal is decomposed into numerous fundamental oscillations, referred to as Intrinsic Mode Functions (IMF). IMF works on two principles :(i) An IMF has only one extremum between two succeeding zero-crossings i. e., the number of zero-crossing points and local extrema varies at most by one. (ii) The local average of the upper and lower envelopes of an IMF must be zero. Even though EMD has been proved to be remarkably suitable for many applications, some drawbacks remain. For instance, EMD is vulnerable to noise, and it can decompose the signal into several components.

A single channel Mirnov coil analysis with SVD+Hilbert Huang Transform (HHT) (Expand HHT) is used to study the MHD activities in a Tokamak [8]. The authors have utilized the SVD to extract the principal axes and principal components. The Empirical Mode Decomposition technique aided to derive the IMFs from the extracted principal axes. The IMFs are then processed using Hilbert Transform for time–frequency analysis of IR-T1 and Golem tokamak data. The proposed technique failed to address the intermittence (noise) problem and the multi-component problem caused during mode mixing. Moreover, the authors utilized a single channel Mirnov coil signal for analyzing the time–frequency behaviour using HHT.

In this proposed work, we investigate how to analyze the magnetic fluctuations caused by MHD activities using the combination of SVD and Multichannel VMD (MVMD). The proposed technique analyzes the Mirnov coil signals for extracting the modes' number and time–frequency identification of dominant MHD. The details of the work are as follows: The proposed technique works in three stages. Initially, SVD is used to obtain the Principal Axes (PA) for spatial structure. The Principal Components (PC) provided the temporal structure. Next, the variational modes decompose the harmonics of MHD utilizing MVMD from the critical PA. Finally, the variational modes extract the time–frequency performance of multichannel Mirnov coils with Hilbert Transform. This analysis aids in studying the time–frequency behaviour of MHD fluctuations.

The rest of this research article is divided into various sections as found below:

In Sect. "Theoretical study", we have briefly introduced the spatial analysis using SVD and the principle of Multichannel-VMD. Section "Experimental results" discusses the Results experimentally. It demonstrates the utilization of the SVD+MVMD technique to analyze the Mirnov coil signal. Initially, simulated artificial time series data is analyzed to see the strength and weaknesses of our work, and based on the knowledge gained, we have analyzed the Mirnov coil data from GOLEM tokamak. Section "Conclusion" Concludes the present some of the significant contributions of the research.

Theoretical Study

Singular Value Decomposition (SVD)

The SVD technique [9] is the most used and widely discussed in research papers that deal with Magnetic fluctuations analysis. One of the advantages of the SVD method is filtering the uncorrelated noise in the data. Consider an NxM rectangular matrix formed with a time series of N-data points of the same physical number at different positions M-channels (i.e., column index = channel and row index = time).

The SVD of the given X is expressed in a product matrix. Where X = VSU^T, where U *is* an MxM orthonormal matrix, V is an NxM orthonormal matrix, and S is an MxM diagonal matrix. The column matrix of U is equivalent to the eigenvectors, and the *SVs* are analogous to eigenvalues. The SVD is equal to the similarity transformation, diagonalized as a square matrix. The *SVs* are the principal components of X and the principal axes are U formed from the column matrix. The *PCs* give the time evolution of signals along with the *PAs*.

The application of the SVD technique to a multichannel time–space matrix of a Mirnov coil is to understand magnetic fluctuations from the singular values and principal axes. The extracted significant modes in the signal from the principal axes correspond to the SVs. Later, the extracted principal axes are plotted with a polar plot, marking the probe positions vs principal axes. In this technique, the mode coupling of signals was a non-sinusoidal eigenvector in space (two or more modes consisting of different mode numbers but with the same frequency components). Two or more modes with the same mode numbers with various frequencies are visualized as single-mode by the polar plot representation.

Multichannel-Variational Mode Decomposition (MVMD)

A recently developed data decomposition technique using a variational approach is termed Variational Mode Decomposition (VMD) [10, 11]. VMD algorithm decomposes a given signal into a discrete number of modes, each having a local harmonic function with slowly varying amplitude and frequency. It is a time–frequency analysis method, where a given input signal *f* is non-recursively decomposed into a small number of bands limited intrinsic modes, known as principal modes. The real-valued input signal '*f*' is decomposed into distinct mode function components f_k, where k = 1, 2, 3……K using a calculus of variations technique. The discrete function f_k is applied to the Fourier transform, which produces a specific sparsity property around the center pulsation frequency $\omega_{k,}$thus satisfying the definition of Intrinsic Mode Function (IMF).

Simultaneously, the Alternating Direction Method Multipliers (ADMM) technique is applied for identifying the central frequency components centered on those frequencies. The sum of the length of the mode's bandwidth is obtained by minimizing the $\Delta\ \omega_k$, (define this notation) via satisfying the criteria that the sum of the mode's bandwidth is equivalent to the given real-valued function f. Instantaneous frequency and bandwidth of these modes are extracted from the decomposed input signal (*f*)'s IMF components. These are represented as $f_k(t) = r_k(t)\ cos\ (\theta_k)(t)$ with physically meaningful amplitude and phase.

After finding the polar parameters for the modes, it is easier to analyze the function *f*, whose complex form is $f_K^A(t) = r_k(t)\ e^{-j\theta_k t}$. Here, f_K^A is the real part, and f_K^H is an imaginary part and the function is analytic for the final complex value, with its spectrum generating positive frequency for real modes, and zero for negative frequencies.

Though the MVMD approach is similar to VMD, it is characteristically different and superior to the univariate VMD technique when applied to multichannel data. Furthermore, the MVMD technique has a divergent approach to univariate VMD in multivariate oscillations extracted from input signals where MVMD checks them in multidimensional space. The generalized VMD method is insignificant as it simply applies VMD individually to channel data distinctly to extract Univariate modulations existing in each channel data. It is incapable of recovering joint information or multivariate oscillations of multiple channels.

The great advantage of MVMD is its ability to align the modes which have similar frequency components across multiple channels. MVMD is well-established in terms of handling mode-alignment property to obtain modulated multivariate oscillations. This paper utilized MVMD for all the PCs obtained from multichannel Mirnov coil data using the SVD technique for extracting the mode functions in multivariate oscillations.

Algorithm.1: Multichannel-VMD approach

Initialize $\{\hat{u}_{k,c}^1\}, \{\omega_k^1\}, \hat{\lambda}_c^1, n \leftarrow 0, \epsilon \leftarrow 10^{-7.}$

Repeat the process for

$n \leftarrow n+1$

For $k = 1{:}K$ do

For $c = 1{:}C$ do *update mode* $\hat{u}_{k,c}$

Updated $\hat{u}_k$ when $\omega \geq 0$:

$$\hat{u}_{k,c}^{n+1}(\omega) \leftarrow \frac{\hat{x_c}(\omega) - \sum_{i<k} \hat{u}_{i,c}^{n+1}(\omega) - \sum_{i>k} \hat{u}_{i,c}^{n}(\omega) + \frac{\hat{\lambda}_c^1(\omega)}{2}}{1 + 2\alpha(\omega - \omega_k^n)^2}$$

End

For $k = 1{:}K$ do

Updating ω_k *center frequency* :

$$\omega_k^{n+1} \leftarrow \frac{\sum_c \int_0^\infty \omega |\hat{u}_{k,c}^{n+1}(\omega)|^2 d\omega}{\sum_c \int_0^\infty |\hat{u}_k^{n+1}(\omega)|^2 d\omega}$$

End

For $c = 1{:}C$ do

Update $\hat{\lambda}_c$

Dual ascent when $\omega \geq 0$

$$\hat{\lambda}_c^{n+1}(\omega) \leftarrow \hat{\lambda}_c^{n}(\omega) + \tau\left(\hat{x_c}(\omega) - \sum_k \hat{u}_{k,c}^{n+1}(\omega)\right)$$

End for

Until convergence: $\Sigma_c \sum_k \frac{\|\hat{u}_{k,c}^{n+1} - \hat{u}_{k,c}^{n}\|_2^2}{\|\hat{u}_{k,c}^{n}\|_2^2} < \epsilon.$

Experimental Results

Study of MHD Activities Using Simulated Data

The proposed technique is first tested using simulated artificial time series data. Simulation is performed to reduce the complexity in understanding the real signals and to clarify the results. This process also displays the strength and weakness of the proposed technique. The artificially simulated time series could be written in the following form,

$$X_{ij} = \frac{1}{\sqrt{N}} \sum_{l} amp_l cos\left(\frac{2\pi m_l}{M}(j-1) + \varnothing_l - 2\pi f_l t_s (i-1)\right) + \eta\sigma_{ij}$$

i.e., the superposition of cosinusoids has the frequency *fl* with rotating mode number *ml* with an amplitude *ampl.* Additionally, it is provided with an added white noise of amplitude. The signal is sensed at M equidistant in poloidal at 2 π and with a sample time Ts. The values of the parameters are: $M = 8$: $T_s = 10^{-3}$ and $N = 1024$ as represented in Table 1.

The script was written in MATLAB R2021a using its inbuilt mathematical functions and performed the pre-processing of the test signal to remove linear trends and mean. The observations from the test signals show that,

1. SVD aids in the extraction of two or more spatial modes with different frequencies and mode numbers into distinct pairs of PAs corresponding to cosine and sine Fourier components.
2. The accuracy of the system increases with an increase in data points rather than the number of periods.
3. Two or additional modes with identical spatial mode numbers but various frequencies are considered a distorted single-mode in time.
4. Two or additional modes consisting of various mode numbers but have similar frequency during rotation, and a constant amplitude is visualized as a single-mode by the SVD with a non-sinusoidal and distorted eigenvector in space. It is due to the diagonal phase relationship that does not vary in time in the covariance matrix (Fig. 1).

Table 1 Parameters used to generate the artificially simulated signal for the analytical test of SVD

m_l	amp_l	f_l	$\varnothing_l$
1.0	1.0	20 + 6.0t	0.00
2.0	0.60	50–1.20t	0.50
3.0	0.20	90–7.60t	0.33
Added noise	0.05	–	–

The SVD analysis is found to be perfect for the numerical test data from the results reproduced from the ref [12].

Spatial–Temporal Analysis of Golem Tokamak (SVD+MVMD)

The collected Signal Golem Tokamak shot number-36593 from the Mirnov coil is carried out for the time window (12.5–13.5 ms) and positioned the GOLEM tokamak at the Faculty of Nuclear Physics and Physical Engineering (FNPPE), the Czech Technical University in Prague [13–15]. It has the exclusive benefit of remote handling via an internet connection. The major radius (R) of the tokamak vessel is 0.4 m. This stainless-steel vessel has a well-equipped poloidal limiter (Molybdenum) with a radius (a) of 0.085 m.

The tokamak is provided with an additional diagnostic system to measure the plasma current, loop voltage, visible emission, toroidal magnetic field, etc. GOLEM is well-equipped with an array of Mirnov coils, bolometers, a visible spectrometer, a fast camera, and 12 Langmuir probes in a radial array. GOLEM tokamak functions at a determined toroidal magnetic field with a range up to 0.8 T. The central electron temperature is less than 100 eV, the maximum plasma density average is $\sim$ 1019 m^{-3}, and the extreme pulse length is around $t < 30$ ms.

SVD analysis is carried out with 4 Mirnov probe coil signals placed around the vacuum chamber. The sampled signal from Mirnov coils is at a 1MHZ sampling rate. Figure 3 represents the signal for the corresponding time (12.5–13.5 ms) on channel-1. To extract the analogous PA and PC of the signal SVD is applied. The singular value (SVs) acquired from the first component has the highest value and is larger than the others.

The polar representation of PA corresponding to the PC is as shown in Fig. 2. The principal components of the first column of the eigenvector are extracted using the MVMD technique for time–frequency analysis, which demonstrated that MHD fluctuations detection is possible from the dominant Principal axes. The first plot is the Intrinsic mode function or mode1 extracted from MVMD of channel-1 data, the spatial structure of the corresponding IMF was plotted. The next one is the IMF of channel data-2b with its corresponding spatial plot is plotted in polar representation.

The principal axes extracted from the SVD and the input signal is plotted as shown in the Fig. 3a. The MVMD identifies the IMF present in the corresponding input signal. The spectral decomposition of the input signal is

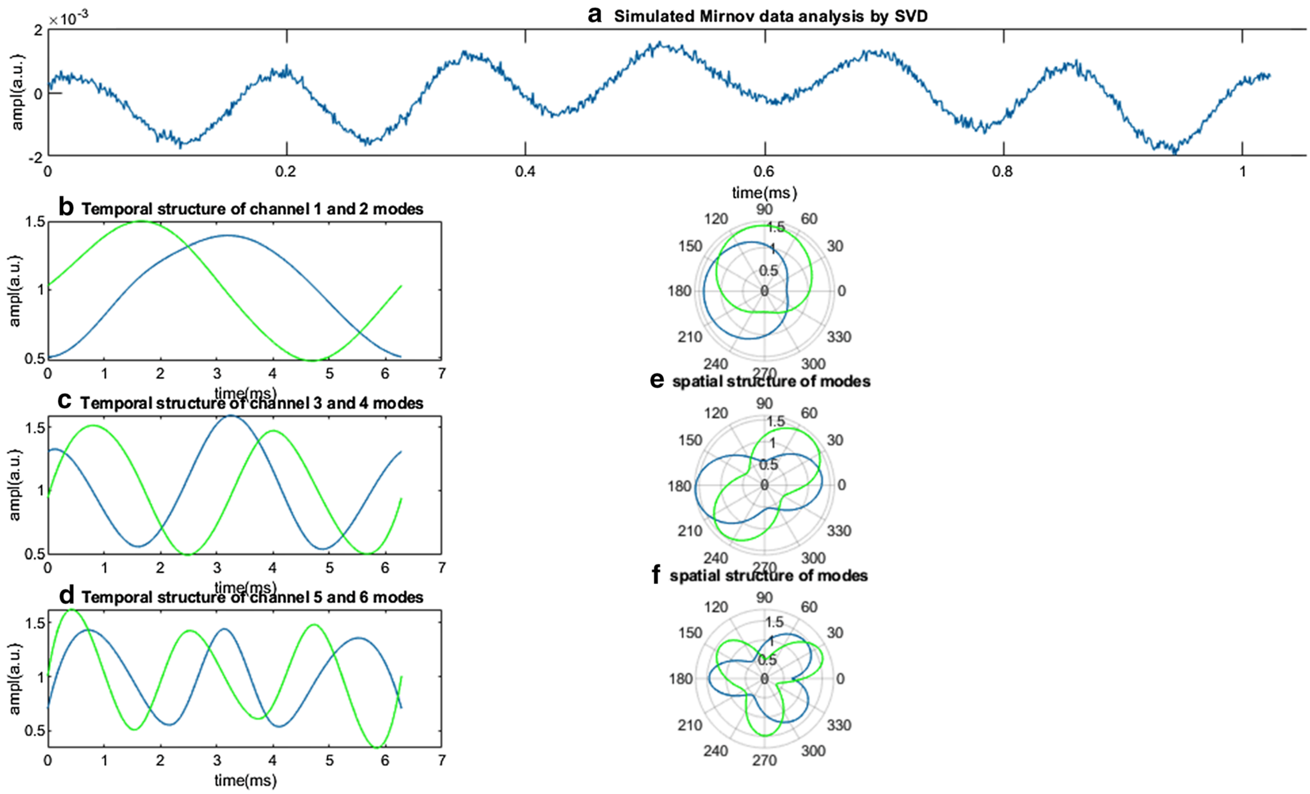

Fig. 1 **a** Simulated Mirnov coil data for SVD, **b** The temporal structure of channel 1 & 2 data, **c** The temporal structure of channel 3 & 4 data, **e** spatial structure of modes obtained from channels 3 & 4, **d** The temporal structure of channel 5 & 6 data and **f**. spatial structure of modes obtained from channels 5 & 6

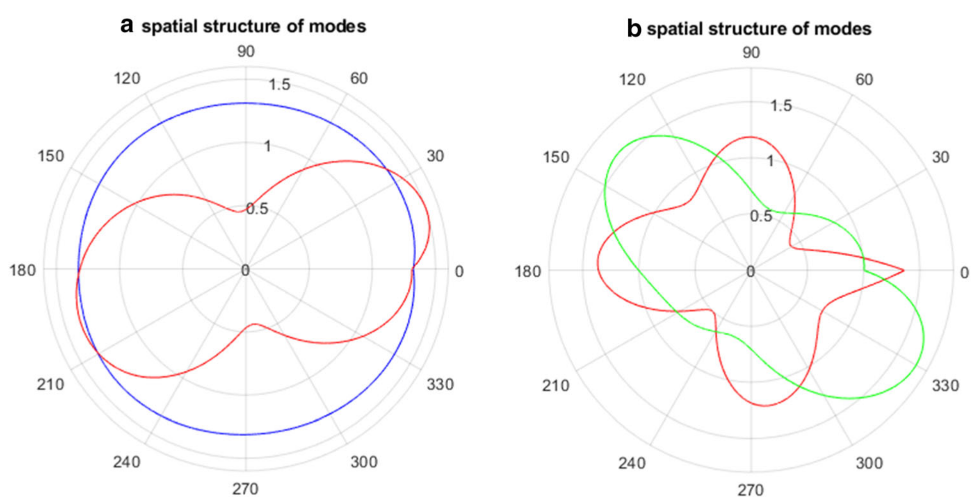

Fig. 2 **a** Spatial structure of channels 1 & 2 and **b** Spatial structure of channels 3 & 4 representing their mode number at corresponding time interval (12.5–13.5 ms) at Mirnov shot 36,593

plotted as shown in the Fig. 3b. Moreover, the spatial plot of channel-1 is plotted in Fig. 2 at 20 kHz, and the spectral estimation plots the corresponding frequency and amplitude present in the magnetic fluctuations of the Mirnov coil signal of channel-1. In the proposed method, the MVMD-spectral decomposition technique has achieved the spatial plot with polar representation and temporal analysis of the time–frequency plot.

Further, the proposed technique was compared with other existing, widely used signal decomposition techniques. The Empirical Mode Decomposition and Time-varying Filter based EMD techniques provide the same set of test signals used in our proposed methodology. The proposed method decomposes the signal faster in terms of time (ms). Also, the decomposition accuracy is calculated based on the addition of decomposed signal difference

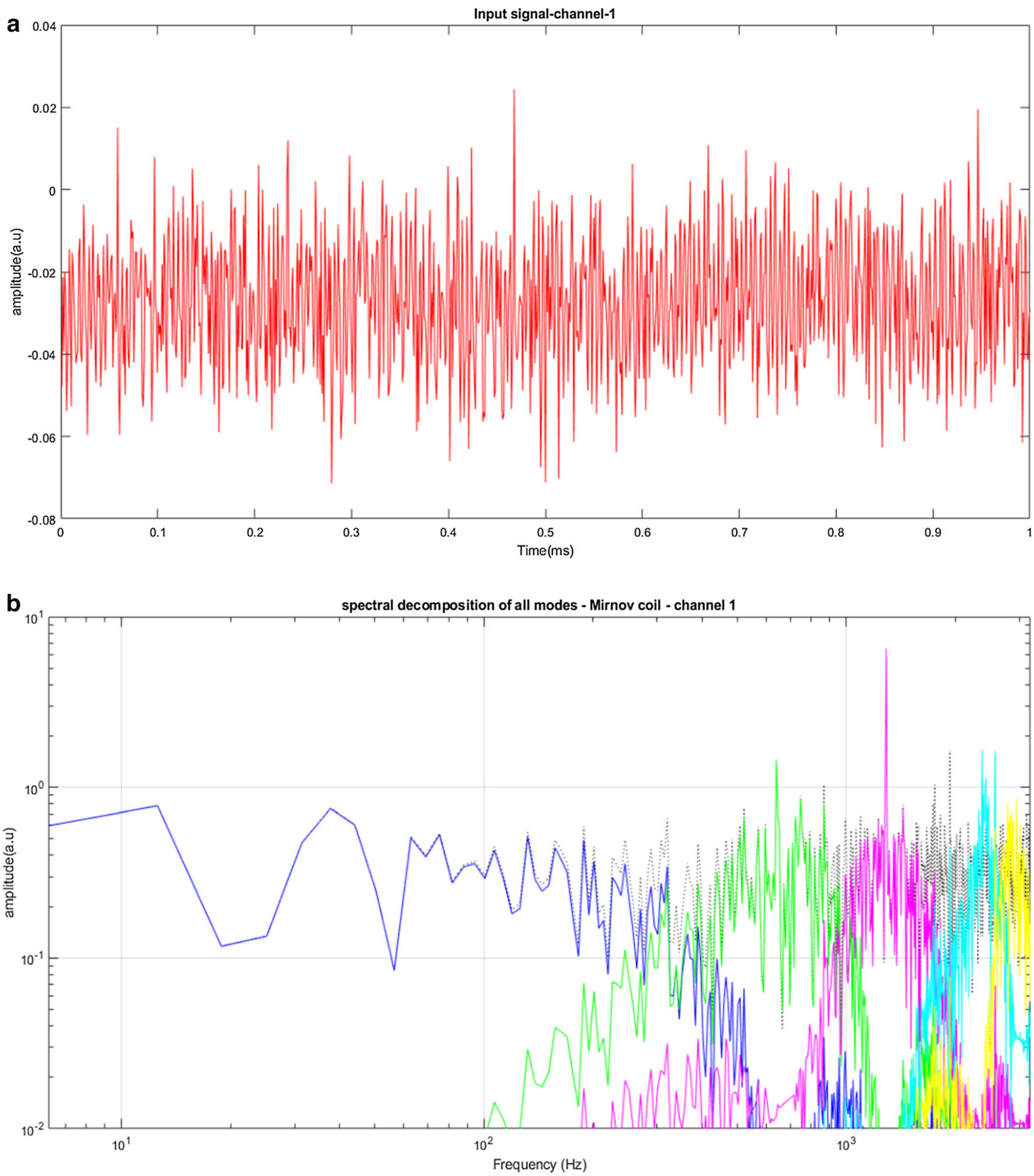

Fig. 3 **a** Input signal of channel-1 and **b** Spectral decomposition of all modes in channel-1

from the original input signal. Figures 4 and 5 depict the performance analysis between the proposed MVMD based decomposition technique is better than the existing methodologies for Mirnov coil decomposition.

Conclusion

The proposed model of combined decomposition technique utilizing SVD+MVMD analysis is effective in decomposing the data for identifying multimode or multivariate MHD fluctuations. In our work, we have applied the SVD technique with minimum time interval data points, which is highly sensitive in identifying the low amplitude in MHD mode. The enormous impact of MVMD results in the separation of amplitude and frequency spectrums from the coupled MHD modes where MVMD was able to extract different amplitude present at various time intervals of MHD mode spectra. SVD provided the Spatial–temporal representation of the structures of Mirnov fluctuations for identifying the mode numbers and dominant frequency in MHD modes with clear peaks of the wave-like modes.

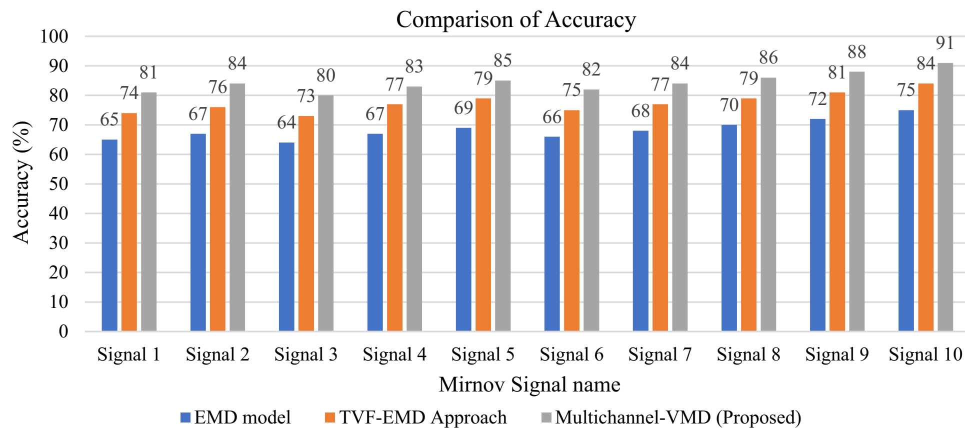

Fig. 4 Comparison of signal decomposition accuracy: EMD vs TVF-EMD vs Proposed MVMD

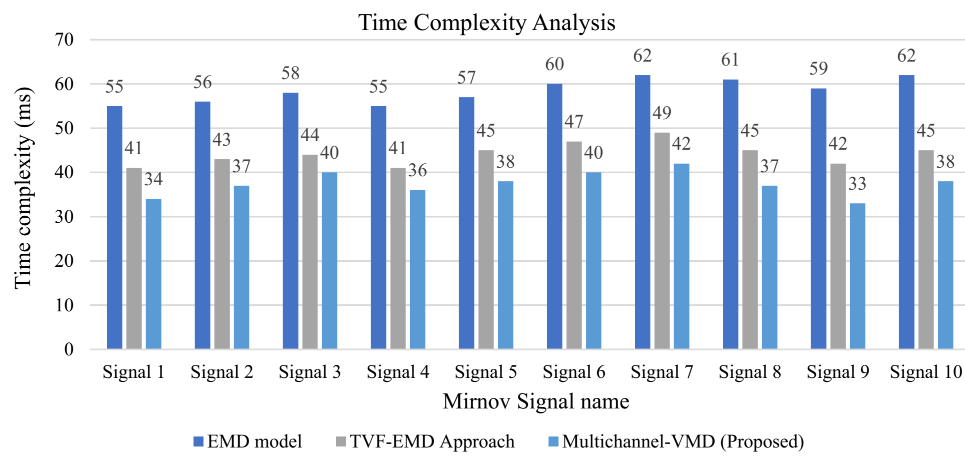

Fig. 5 Time complexity for decomposition: EMD vs TVF-EMD vs Proposed MVMD

From the observation, and there is a high correlation value with a little poloidal separation between magnetic probes that indicate an enormous mode. Also, the amplitude and frequency phase of MHD mode can be analyzed using the spectral decomposition function.

Acknowledgements The authors would like to thank Golem Tokamak team. Authors expresses their special thanks to Dr.V. Svoboda for providing access to Golem Tokamak shot database http://golem.fjfi.cvut.cz/shots/0/.

References

1. L.E. Zakharov, S.A. Galkin, S.N. Gerasimov, Phys Plasmas **19**(5), 055703 (2012)
2. A.H. Boozer, Phys. Plasmas **19**(5), 058101 (2012)
3. A.C.A. Figueiredo, M.F.F. Nave, Nucl. Fusion **44**, L17 (2004)
4. R. Coelho, D. Alves, C. Silva, Rev. Sci. Instrum. **77**, 19 (2006)
5. N.E. Huang, Z. Shen, S.R. Long, M.C. Wu, H.H. Shih, Proc. R. Soc. London A **454**, 903 (1998)
6. R. Jha, D. Raju, A. Sen, Phys. Plasmas **13**, 1 (2006)
7. N.V. Ivanov, A.M. Kakurin, I.I. Orlovskiy, Irregularity of the magnetic island rotation under external helical magnetic perturbation in T-10 tokamak. In Proc. 32nd EPS Conf. on Plasma Physics, vol. 29 (2005)
8. H. Faridyousefi, M.K. Salem, M. Ghoranneviss, J. Fusion Energy **39**, 512 (2020)
9. D. Raju, R. Jha, P.K. Kaw, S.K. Mattoo, Y.C. Saxena, Pramana-J. Phys. **55**, 727 (2000)
10. N.U. Rehman, H. Aftab, IEEE Trans. Sig Process. **67**, 6039 (2019)
11. K. Dragomiretskiy, D. Zosso, IEEE Trans. Signal Process **62**, 531 (2014)
12. C. Nardone, Plasma Phys. Control. Fusion **34**, 1447 (1992)
13. O. Grover, J. Kocman, M. Odstrcil, T. Odstrcil, M. Matusu, J. Stöckel, V. Svoboda, G. Vondrasek, J. Zara, Fusion Eng Des **112**, 1038 (2016)
14. V. Svoboda, A. Dvornova, R. Dejarnac, M. Prochazka, S. Zaprianov, R. Akhmethanov, M. Bogdanova, M. Dimitrova, Z. Dimitrov, O. Grover, L. Hlavata, K. Ivanov, K. Kruglov, P. Marinova, P. Masherov, A. Mogulkin, J. Mlynar, J. Stockel, A. Volynets, J. Phys. Conf. Ser. **768**, 012002 (2016)
15. V. Svoboda, B. Huang, J. Mlynář, G.I. Pokol, J. Stöckel, G. Vondrášek, Fusion Eng. Des. **86**, 1310 (2011)