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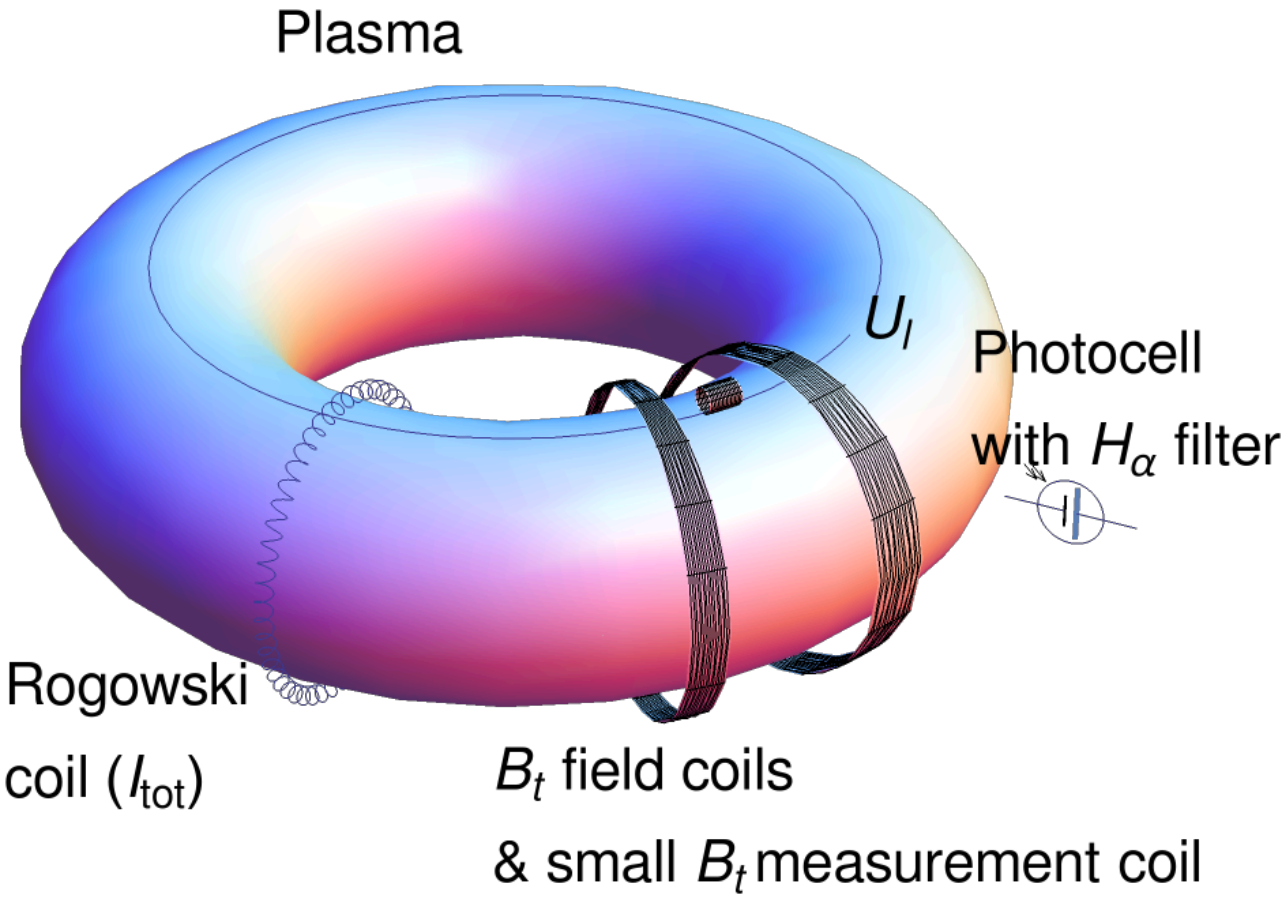
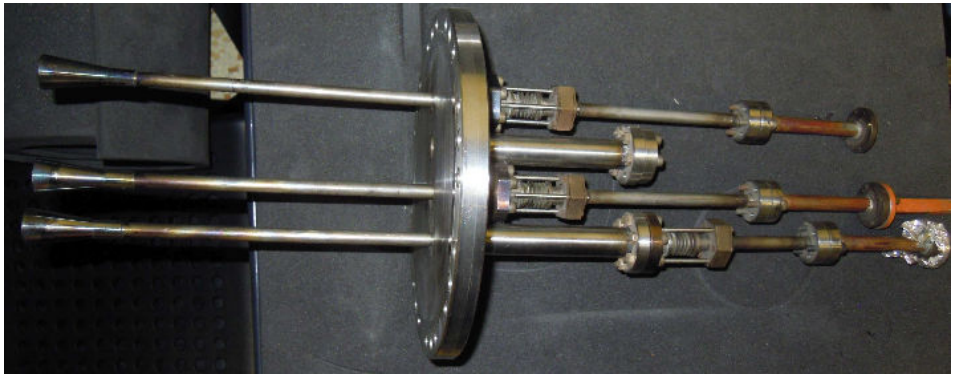
Hands on the GOLEM tokamak - equipment

The GOLEM tokamak - standard diagnostics



The GOLEM tokamak interferometry HW



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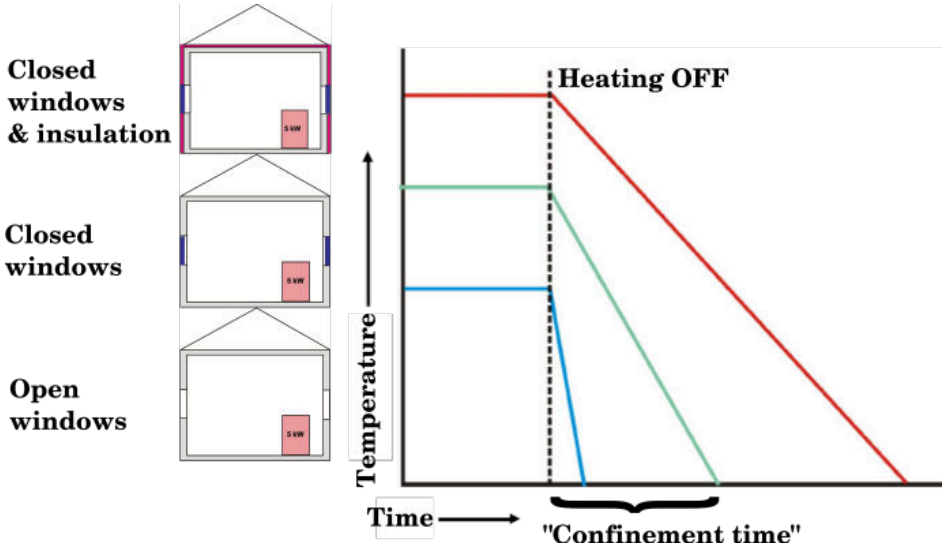
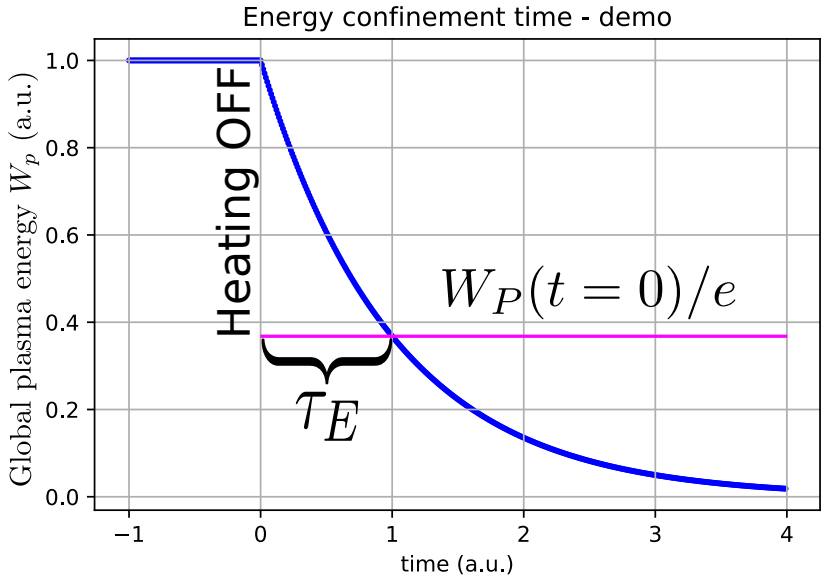
Towards the Electron Energy Confinement Time $\tau_{E,e}$ @tG

House
(cooling after heating stops)



Tokamak
(cooling after heating stops)

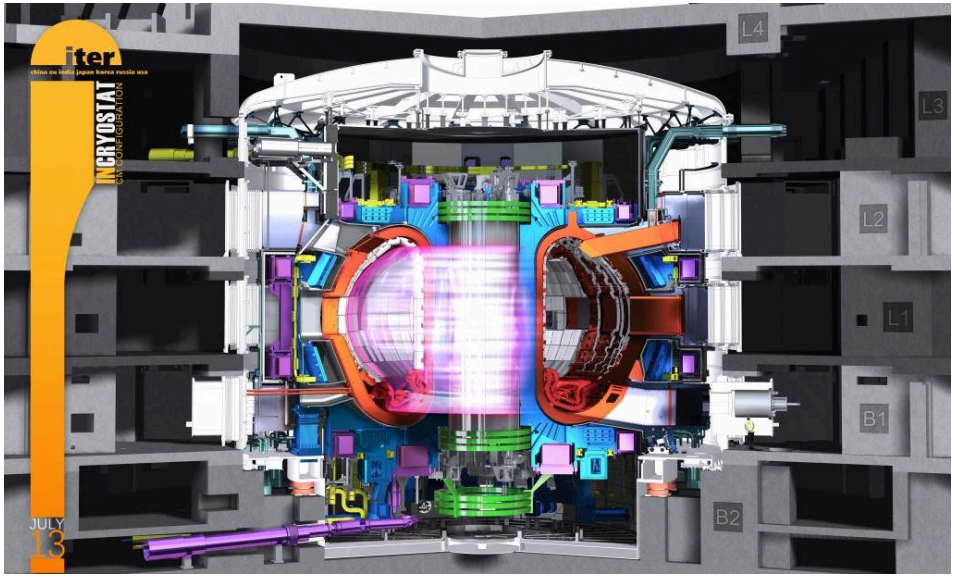


When heating is switched off, both systems lose their internal energy exponentially.
This motivates the definition of the electron energy confinement time $\tau_{E,e}$.

ITER vs GOLEM

ITER:

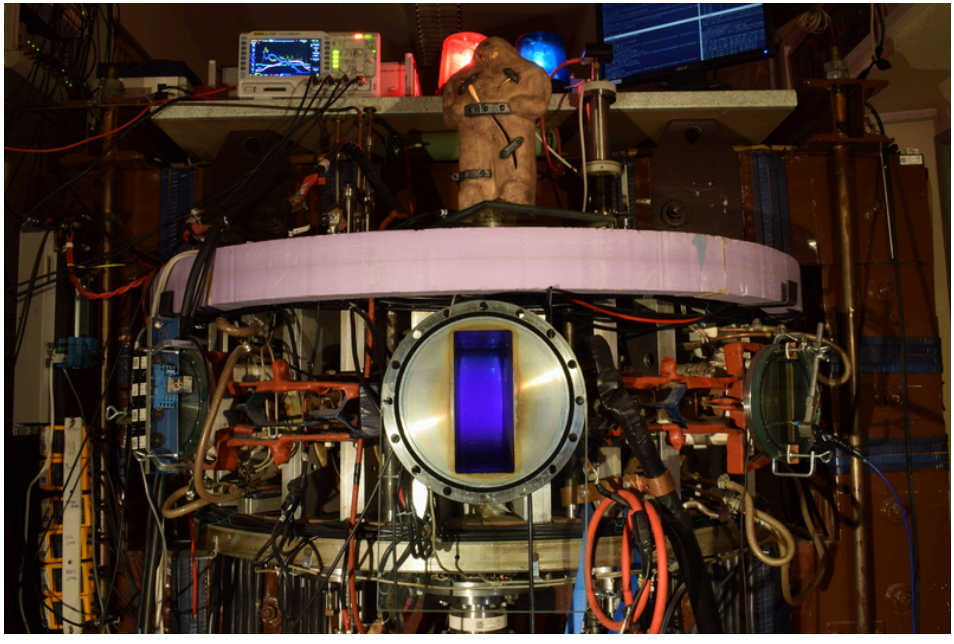
$\tau_E \approx 3.6$ s



credit:[?]

GOLEM:

$\tau_{E,e} \approx$ on the order of 10 μs



credit:[?]

Different scale, same physics — and you will measure it today @tG.

What follows — roadmap

- **Lawson criterion** — the fusion requirement
- **Quasi–stationary method @tG** — how we measure confinement
- **Ohmic heating power** P_{OH}
- **Electron temperature** from Spitzer conductivity T_e
- **Electron energy content** W_e
- **Putting it all together:** the confinement time $\tau_{E,e}$

Lawson criterion

credit:Lawson criterion @ Wiki [?]

- To achieve net fusion power, fusion heating must exceed losses:

$$P_{\text{fusion}} \geq P_{\text{loss}}.$$

- Plasma energy content:

$$W = 3nk_{\text{B}}T.$$

- Confinement time links energy to losses:

$$\tau_E = \frac{W}{P_{\text{loss}}}.$$

- Fusion reaction rate (DT):

$$f = \frac{1}{4}n^2\langle\sigma v\rangle, \qquad P_{\text{fusion}} = f\,E_\alpha, \qquad E_\alpha = 3.5\ \text{MeV}.$$

- Condition for ignition becomes the Lawson criterion:

$$n\tau_E \geq \frac{12}{E_\alpha}\frac{k_{\text{B}}T}{\langle\sigma v\rangle} \approx 1.5\times 10^{20}\,\text{s/m}^3 \quad (\text{DT at } T \approx 26\ \text{keV}).$$

Measuring $\tau_{E,e}$ @tG: the quasi–stationary method

- On GOLEM we cannot switch off the ohmic heating abruptly (capacitor discharge drives the current throughout the pulse).
- Therefore we use the **quasi–stationary phase** of the discharge:

$$\frac{dI_p}{dt} \approx 0, \qquad \frac{dT_e}{dt} \approx 0, \qquad \frac{dW_e}{dt} \approx 0.$$

- In this regime, the ohmic heating balances the losses:

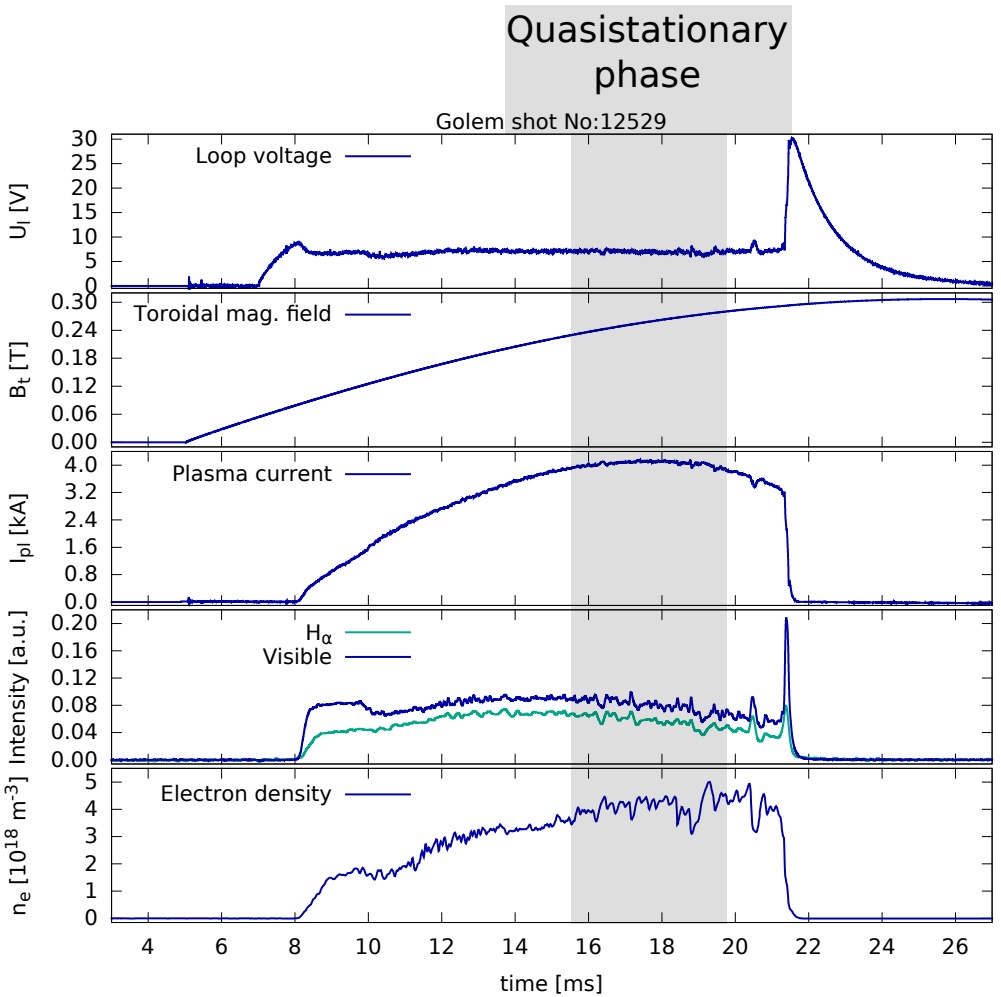
$$P_{\mathrm{OH}} \approx P_{\mathrm{loss}}.$$

- The electron energy confinement time becomes:

$$\tau_{E,e} \approx \frac{W_e}{P_{\mathrm{OH}}}.$$

- This provides a consistent and reproducible estimate of $\tau_{E,e}$ suitable for a small tokamak like GOLEM.

The discharge — quasi–stationary phase



Quasi–stationary phase:

- Plasma current I_p, electron temperature T_e and density n_e vary only slowly.
- Therefore $dW_e/dt \approx 0$.
- Ohmic heating balances losses:

$$P_{\mathrm{OH}} \approx P_{\mathrm{loss}}.$$

- This is the interval used to estimate the electron energy confinement time:

$$\tau_{E,e} \approx \frac{W_e}{P_{\mathrm{OH}}}.$$

- All quantities are measurable @tG.

Ohmic heating power P_{OH} @tG

On the GOLEM tokamak the plasma is heated solely by **ohmic heating**, driven by the toroidal electric field that sustains the plasma current I_p. The instantaneous ohmic heating power is directly obtained from two measurable quantities:

$$P_{\mathrm{OH}}(t) = V_{\mathrm{loop}}(t)\, I_p(t),$$

where V_{loop} is the toroidal loop voltage measured by the flux loop, and $I_p(t)$ is the plasma current from the Rogowski coil. In the quasi–stationary phase, where $dW_e/dt \approx 0$, this power balances the energy losses:

$$P_{\mathrm{OH}} \approx P_{\mathrm{loss}},$$

providing the key ingredient for estimating the electron energy confinement time $\tau_{E,e}$ @tG.

Central electron temperature T_e (Spitzer conductivity)

The time evolution of the central electron temperature $T_e(0,t)$ is obtained from Spitzer's resistivity formula for a collisional, fully ionized plasma.

- Spitzer resistivity (symbolically):

$$\eta(T_e) \propto Z_{\mathrm{eff}} \ln\Lambda \, T_e^{-3/2}.$$

- Using $\eta = R_p A/L$ with plasma resistance $R_p(t)$ one may write for the on–axis temperature:

$$T_e(0,t) = \left(\frac{R_0}{a^2} \frac{8 Z_{\mathrm{eff}}}{1544\, R_p(t)} \right)^{2/3} \qquad [\mathrm{eV};\ m, \Omega].$$

- For the particular case of the GOLEM tokamak (fixed geometry, $Z_{\mathrm{eff}} \approx 1$, $\ln\Lambda \approx \mathrm{const}$) all geometric and numerical factors can be grouped into a single coefficient, giving the practical formula:

$$T_e(0,t) \approx 0.9 \left(\frac{I_p(t)}{V_{\mathrm{loop}}(t)} \right)^{2/3} \qquad [\mathrm{eV};\ \mathrm{A}, \mathrm{V}].$$

- Thus @tG the central electron temperature can be estimated directly from the measured $I_p(t)$ and $V_{\mathrm{loop}}(t)$.

Electron energy content W_e @tG

The electron energy content follows from the ideal-gas expression for a single thermal species:

$$W_e(t) = \frac{3}{2}\, n_e(t)\, k_{\mathrm{B}}\, T_e(t)\, V_p,$$

where $n_e(t)$ is the line-averaged (or central) electron density, $T_e(t)$ the central electron temperature, and V_p the plasma volume. For a circular, low–aspect–ratio tokamak the plasma volume is well approximated by:

$$V_p \approx 2\pi^2 R_0 a^2,$$

with R_0 the major radius and a the minor radius. At GOLEM all quantities entering W_e are experimentally accessible:

- $n_e(t)$ from microwave interferometry,
- $T_e(t)$ from Spitzer conductivity,
- V_p from known machine geometry.

Thus $W_e(t)$ can be evaluated directly during the quasi–stationary phase of the discharge.

Electron energy confinement time $\tau_{E,e}$ @tG

In the quasi–stationary phase of the discharge, where $dW_e/dt \approx 0$, the ohmic heating balances the energy losses:

$$P_{\mathrm{OH}} \approx P_{\mathrm{loss}}.$$

The electron energy confinement time is therefore obtained as:

$$\tau_{E,e}(t) = \frac{W_e(t)}{P_{\mathrm{OH}}(t)} = \frac{\frac{3}{2}\, n_e(t) k_{\mathrm{B}}\, T_e(t) V_p}{V_{\mathrm{loop}}(t)\, I_p(t)}.$$

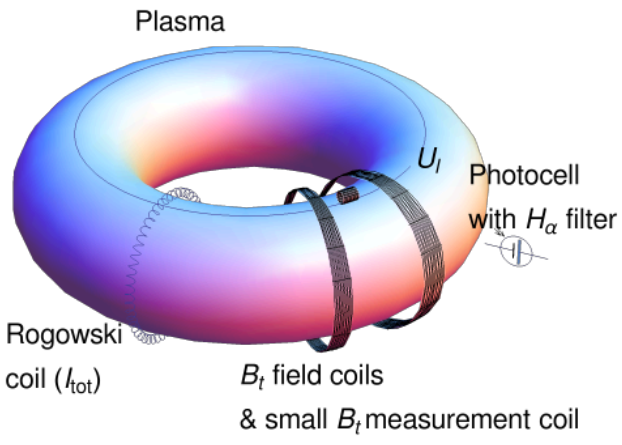
All entering quantities are experimentally accessible @tG:

- $P_{\mathrm{OH}}(t) = V_{\mathrm{loop}}(t)\, I_p(t)$,
- $T_e(t)$ from Spitzer conductivity,
- $n_e(t)$ from interferometry,
- V_p from machine geometry.

This yields a self–consistent, fully diagnostic–based estimate of the **electron energy confinement time** on the GOLEM tokamak.

Measured quantities @tG for computing $\tau_{E,e}$

Directly measured on GOLEM:



Diagnostics relevant to $\tau_{E,e}$ on GOLEM

- **Plasma current** $I_p(t)$ from the Rogowski coil (shown as the helical pickup).
- **Loop voltage** U_l (not needed for q, but measured).
- **Machine geometry:** major radius R_0 and minor radius a.
- **Electron density** from interferometry.

These are all required inputs to compute:

$$\tau_{E,e}(t) = \frac{W_e(t)}{P_{\mathrm{OH}}(t)} = \frac{\frac{3}{2}\, n_e(t) k_{\mathrm{B}}\, T_e(t) V_p}{V_{\mathrm{loop}}(t)\, I_p(t)}.$$

From measured signals to $\tau_{E,e}$ @tG

Vacuum discharge
$I_{\mathrm{ch}}^{\mathrm{vac}}(t),\ V_{\mathrm{loop}}^{\mathrm{vac}}(t)$

Plasma discharge
$I_{\mathrm{ch+p}}(t),$
$V_{\mathrm{loop}}(t),\ n_e(t)$

Chamber resistance
$$R_{\mathrm{ch}} \approx \frac{V_{\mathrm{loop}}^{\mathrm{vac}}}{I_{\mathrm{ch}}^{\mathrm{vac}}}$$

Plasma current
$$I_p = I_{\mathrm{ch+p}} - \frac{V_{\mathrm{loop}}}{R_{\mathrm{ch}}}$$

Quasi–stationary interval $[t_1, t_2]$:
$$\bar{I}_p = \langle I_p \rangle, \quad \bar{V}_{\mathrm{loop}} = \langle V_{\mathrm{loop}} \rangle, \quad \bar{n}_e = \langle n_e \rangle$$

Plasma resistivity
$$R_p = \frac{\bar{V}_{\mathrm{loop}}}{\bar{I}_p}$$

Central T_e (Spitzer)
$$T_e(0) \approx 0.9\, R_p^{-2/3}$$

Ohmic heating
$$\bar{P}_{\mathrm{OH}} = \bar{V}_{\mathrm{loop}}\, \bar{I}_p$$

Electron energy
$$\bar{W}_e = \tfrac{3}{2}\, \bar{n}_e k_{\mathrm{B}}\, T_e V_p$$

Electron energy confinement time
$$\tau_{E,e} = \frac{\bar{W}_e}{\bar{P}_{\mathrm{OH}}}$$

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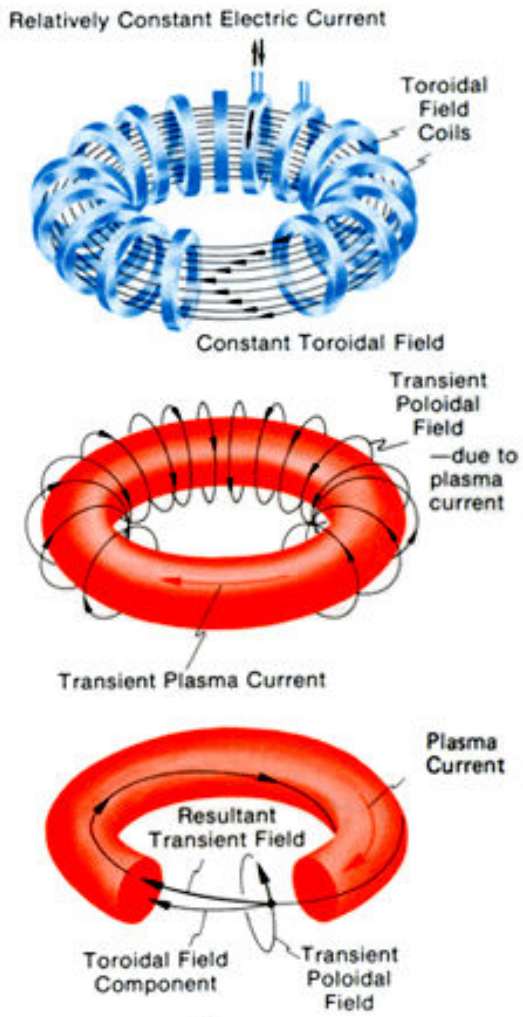
Towards the Safety Factor q



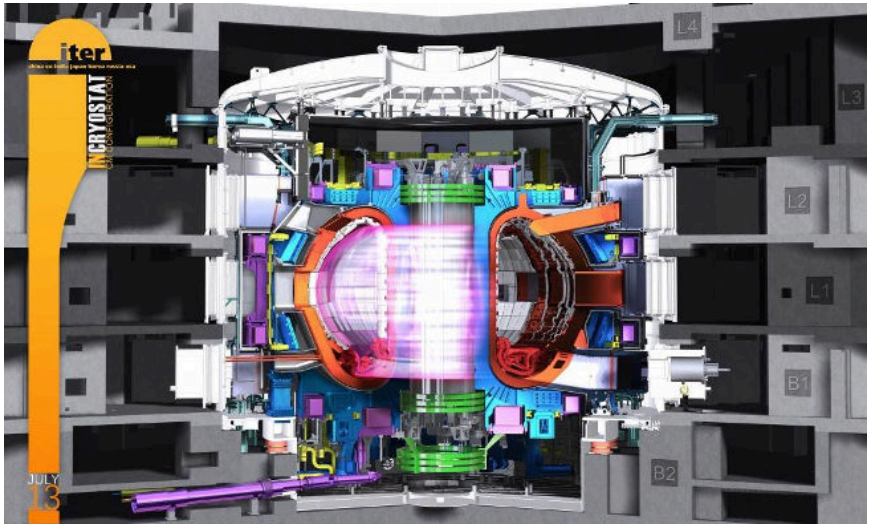
Magnetic field lines in a tokamak

Why it matters:

- q measures how much field lines *twist* around the torus.
- A key parameter controlling MHD stability.
- Rational q surfaces ($q = m/n$) $\rightarrow$ possible island formation.
- Operating limits: too low $q \rightarrow$ kink instabilities.
- ITER, JET, ASDEX-U all use $q(a)$ as a primary equilibrium metric.
- **Fundamental limits:** $q(0) > 1$ (avoid internal kink), $q(a) \gtrsim 2$ (avoid external kink).

ITER vs GOLEM: typical values of $q(a)$

ITER

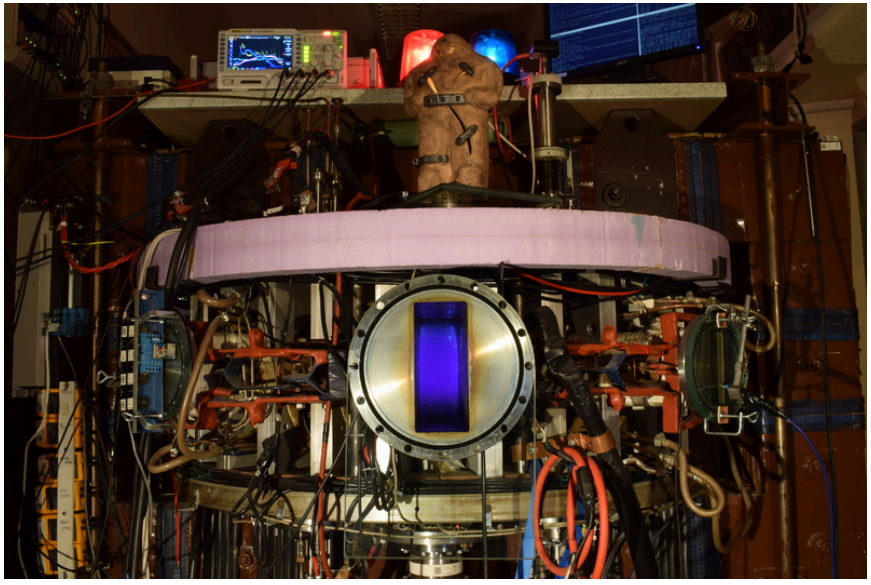


Typical edge safety factor:

$$q(a) \sim 3 - 4$$

Large device, strong shaping, high stability margin.

GOLEM



Typical edge safety factor:

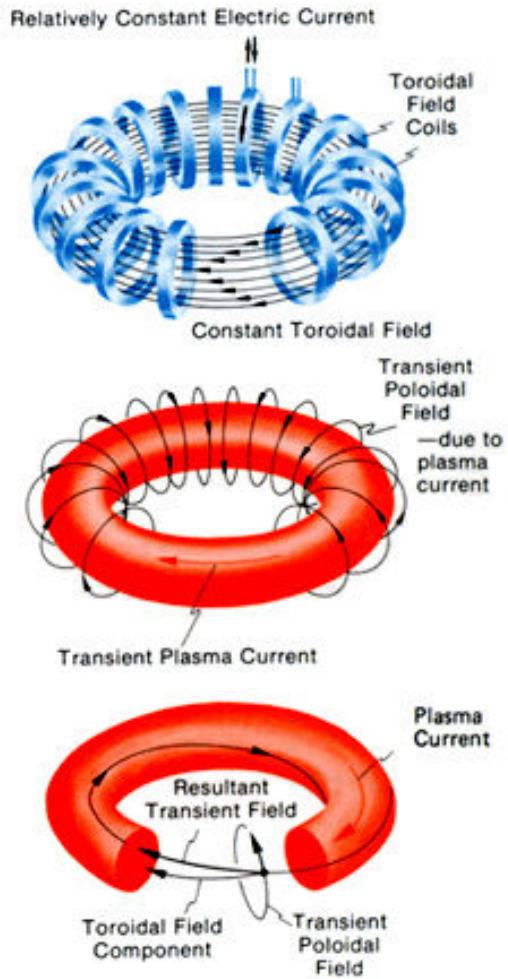
$$q(a) \sim 2 - 6$$

Small device: circular plasma, flexible operating space.

What follows — roadmap to the safety factor q

- **Definition of the safety factor** — toroidal/poloidal twist of magnetic field lines.
- **Circular tokamak geometry** — relation between B_t, B_p and plasma current.
- **Quantities measured @tG** — $I_p(t)$ from Rogowski, B_t from toroidal field coils, geometry (R_0, a).
- **Computing the edge safety factor** — $q(a) = \dfrac{2\pi a^2 B_t}{\mu_0 R_0 I_p}$.
- **Typical operating window** — GOLEM: $2 < q(a) < 6$, stability limits $q(0) > 1$, $q(a) \gtrsim 2$.

Definition of the Safety Factor q



Magnetic field line winding

Geometric definition:

$$q(r) = \frac{\text{toroidal turns}}{\text{poloidal turns}}$$

In a circular tokamak:

$$q(r) = \frac{rB_t}{R_0 B_p(r)}$$

where B_t is the toroidal field, $B_p(r)$ the poloidal field, R_0 the major radius. Using Ampère's law:

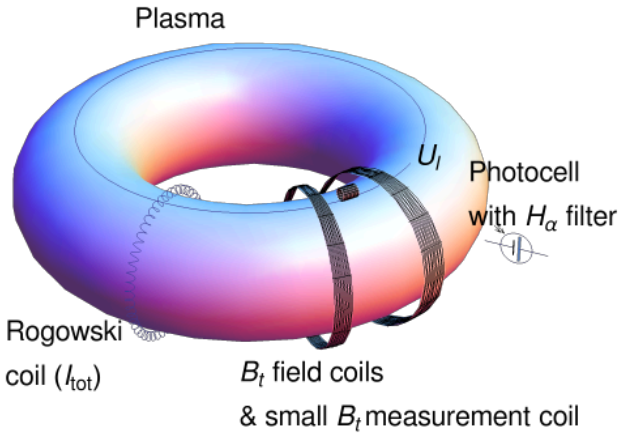
$$B_p(r) = \frac{\mu_0 I_p(r)}{2\pi r}$$

This gives the practical form:

$$q(r) = \frac{2\pi r^2 B_t}{\mu_0 R_0 I_p(r)}.$$

Measured quantities @tG for computing $q(a)$

Directly measured on GOLEM:



Diagnostics relevant to $q(a)$ on GOLEM

- **Plasma current** $I_p(t)$ from the Rogowski coil (shown as the helical pickup).
- **Toroidal field** B_t from the TF coil current (and verified by the small Bt measurement coil).
- **Machine geometry:** major radius R_0 and minor radius a.

These are all required inputs to compute:

$$q(a) = \frac{2\pi a^2 B_t}{\mu_0 R_0 I_p}.$$

Computing the edge safety factor $q(a)$

Definition:

$$q(r) = \frac{rB_t}{R_0 B_p(r)}.$$

Using Ampère's law at the plasma edge:

$$B_p(a) = \frac{\mu_0 I_p}{2\pi a}.$$

Insert into the definition:

$$q(a) = \frac{2\pi a^2 B_t}{\mu_0 R_0 I_p}.$$

For GOLEM (fixed geometry):

$$K_{\mathrm{geom}} = \frac{2\pi a^2}{\mu_0 R_0}$$

$$q(a) = K_{\mathrm{geom}} \frac{B_t}{I_p}.$$

Thus:measuring $I_p(t)$ and knowing B_t is fully sufficient to compute $q(a)$ for any discharge on GOLEM.

From measured signals to the safety factor $q(a)$ @tG

Plasma current
$I_p(t)$ from Rogowski

Toroidal field
B_t from TF coils

GOLEM geometry (constant)
$R_0,\ a$

Quasi–stationary interval
and mean current
$\bar{I}_p \ = \ \langle I_p \rangle$

Safety factor at the edge:

$$q(a) \ = \ \frac{2\pi a^2 B_t}{\mu_0 R_0 \bar{I}_p}$$